BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2026

Senior Preliminary Problems & Solutions

1. The sum of the numerical coefficients in the complete expansion of $(x^2 - 2xy + y^2)^7$ is:

   (**A**∗) 0 (B) 7 (C) 14 (D) 128 (E) 128^2

   Solution

   The sum of the numerical coefficients in the expansion of $(x^2 - 2xy + y^2)^7 = (x - y)^{14}$ equals the given expression with $x = 1$ and $y = 1$. That is $(1 - 1)^{14} = 0$. The answer is 0.

   Answer: A

2. Jenny chose three different nonzero digits. Owen wrote down all the nine possible two-digit integers obtained from the digits chosen by Jenny. (For example if Jenny chooses 4, 7 and 9 then Owen writes 44, 47, 49, 74, 77, 79, 94, 97 and 99.) Assume that the sum of the integers written by Owen equals 231. The product minus the sum of the digits chosen by Jenny equals

   (A) 5 (**B**∗) 1 (C) 8 (D) 4 (E) 12

   Solution

   Let a, b and c be the digits chosen by Jenny. Each digit was used three times in each decimal place. Therefore, the sum of the written integers equals

   $$3 \cdot 10 \cdot (a + b + c) + 3 \cdot (a + b + c) = 231$$

   which implies $a + b + c = 7$. Without loss of generality, assume that $a < b < c$. Then $a = 1$ because otherwise $2 + 3 + 4 > 7$. So, $b + c = 6$ and therefore $b = 2$ and $c = 4$. Then the product minus the sum of these digits equals $1 \cdot 2 \cdot 4 - (1 + 2 + 4) = 1$.

   Answer: B

3. Define functions f and g as follows: for every real numbers a and b, $f(a, b) = 3a + b$ and $g(a, b) = a - 2b$. Find $g(f(3, 4), f(4, 3))$.

   (A) 8 (B) -4 (C) 0 (D) 12 (**E**∗) -17

   Solution

   $f(3, 4) = 3 \cdot 3 + 4 = 13$ and $f(4, 3) = 3 \cdot 4 + 3 = 15$. Then

   $$g(f(3, 4), f(4, 3)) = g(13, 15) = 13 - 2 \cdot 15 = -17.$$

   Answer: E

4. Find the number of distinct integer solution triples (x, y, z) (positive or negative) of the equation

   $$\frac{xy}{z} + \frac{xz}{y} + \frac{yz}{x} = 3.$$

(A) 0 (B) 1 (C) 2 (**D***) 4 (E) 5

Solution

Multiply each side of the equation

$$\frac{xy}{z} + \frac{xz}{y} + \frac{yz}{x} = 3$$

by xyz.

We obtain

$$x^2y^2 + x^2z^2 + y^2z^2 = 3xyz.$$

The left side is always positive (note, x, y and z cannot be 0 as they are initially in the denominators). The integer solutions are:

- $x = 1, y = 1, z = 1$
- $x = 1, y = -1, z = -1$
- $x = -1, y = 1, z = -1$
- $x = -1, y = -1, z = 1$

The answer is 4.

Answer: D

5. Recall that for a positive integer n, one defines $n!$ by

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$

where $n!$ is called n factorial. For example, $3! = 1 \times 2 \times 3 = 6$ and $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$. Determine the remainder when $1! + 2! + 3! + \cdots + 2026!$ is divided by 7.

(A) 0 (B) 1 (C) 2 (D) 4 (**E***) 5

Solution

Note that $7! + 8! \cdots + 2026!$ is divisible by 7. So, we only need to calculate $(1! + \cdots + 6!) \mod 7$. We obtain

$$1! + \cdots + 6! = 1 + 2 + 6 + 24 + 120 + 720 = 873$$

and $873 \mod 7 = 5$.

Answer: E

6. Five people are accused of a robbery, but only one is guilty. Each gives a statement, but exactly two of the statements are false. This is enough to give a decisive answer. What single person is guilty?

Alex: It was Ben.
Ben: Alex is lying.
Cindy: Dennis is innocent.
Dennis: Ellie is innocent.
Ellie: Dennis is telling the truth.

(A) Alex (B) Ben (C) Cindy (**D**∗) Dennis (E) Ellie

Solution

Denote the statements given by Alex, Ben, Cindy, Dennis and Ellie by A, B, C, D and E, respectively. The following table shows the truth values of these five statements corresponding to each of the five possibilities when exactly one person is guilty.

	A	B	C	D	E
Alex is guilty	False	True	True	True	True
Ben is guilty	True	False	True	True	True
Cindy is guilty	False	True	True	True	True
Dennis is guilty	False	True	False	True	True
Ellie is guilty	False	True	True	False	False

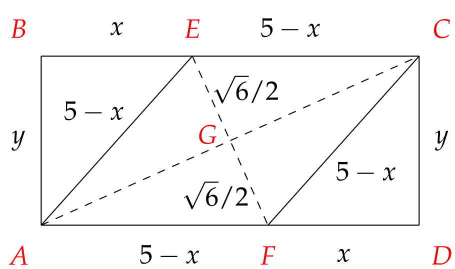
Exactly two false statements appear only when Alex and Cindy are lying. So, Dennis is guilty.

Answer: D

7. The length of a rectangle is 5 and its width is less than 4. The rectangle is folded so that two diagonally opposite vertices coincide. If the length of the crease (fold) is $\sqrt{6}$, then the width of the rectangle is:

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (**C**∗) $\sqrt{5}$ (D) 2 (E) $\sqrt{\frac{11}{2}}$

Solution



Consider the rectangle $ABCD$ with length $AD = BC = 5$ and width $AB = CD < 4$. We fold it in such a way that the vertices A and C coincide and the length of the segment EF (the crease) equals $\sqrt{6}$. This results in a rhombus $AECF$ as in the diagram. Let $BE = x$ and $AB = y$. Then the side of the rhombus $AECF$ has length $5 - x$. We are going to find the width of the rectangle, y, by forming a system of equations.

By the Theorem of Pythagoras,

- from the right triangle ABE, $x^2 + y^2 = (5 - x)^2$
- from the right triangle EGC, $GC^2 = (5 - x)^2 - (\sqrt{6}/2)^2$
- from the right triangle ADC, $GC^2 = \frac{1}{4}(5^2 + y^2)$

We obtain the following system of equations:

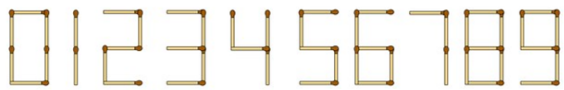
$$\begin{cases} x^2 + y^2 = (5 - x)^2 \\ (5 - x)^2 - \frac{6}{4} = \frac{1}{4}(5^2 + y^2) \end{cases}$$

$$\begin{cases} y^2 = 25 - 10x \\ 4(5-x)^2 - 6 = 50 - 10x \end{cases} \qquad \begin{cases} y^2 = 25 - 10x \\ 4x^2 - 30x + 44 = 0 \end{cases}$$

$x = \frac{30 \pm 14}{8} = 5.5$ or 2. Since the length of the rectangle is 5 only $x = 2$ is appropriate. Then the width of the rectangle equals $y = \sqrt{25 - 10 \cdot 2} = \sqrt{5}$.

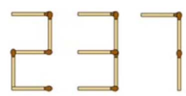
Answer: C

8. Matchsticks can form each of the ten digits as follows:



How many 3-digit odd numbers with distinct digits can be formed using an odd number of matchsticks in total?

For example



is a 3-digit odd number with distinct digits formed using an odd number of matchsticks.

(A) 220 (B) 92 (C) 116 (D) 184 (**E**$*$) 154

Solution

Digit	# of sticks
0	6
1	2
2	5
3	5
4	4
5	5
6	6
7	3
8	7
9	6

For an odd three-digit number the first digit cannot be 0 and the last digit must be odd.

In order to use an odd number of sticks we must have either all the three digits with an odd number of sticks each or exactly two digits with an even number of sticks and one with an odd number of sticks.

Denote by E, respectively, O a digit formed with an even, respectively, odd number of matchsticks.

Then $E:\ 0, 1, 4, 6, 9$ and $O:\ 2, 3, 5, 7, 8$.

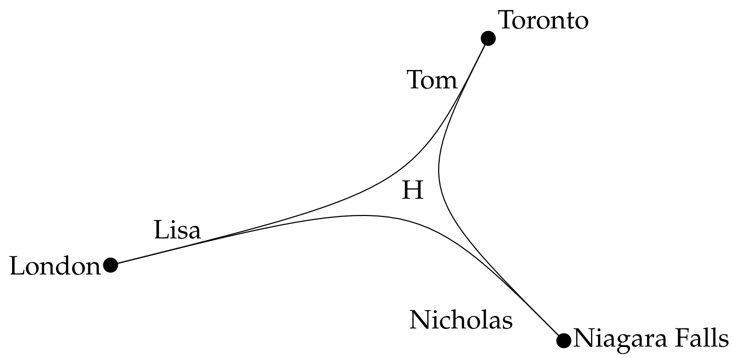
We obtain the following possibilities without repeating the digits:

Number	# of ways
EEO	$4 \cdot 4 \cdot 3 = 48$
EOE	$3 \cdot 5 \cdot 2 = 30$
OEE	$5 \cdot 4 \cdot 2 = 40$
EEO	$3 \cdot 4 \cdot 3 = 36$

So, there are $48 + 30 + 40 + 36 = 154$ of such numbers in total.

Answer: E

9. Rail lines through Hamilton (H) connect London, Toronto, and Niagara Falls as shown. In London, Lisa counts 190 incoming and outgoing trains in total. Similarly, Tom counts 208 trains in Toronto, and Nicholas counts 72 trains in Niagara Falls. Assuming that no train starts, ends, or reverses direction in Hamilton, how many trains traveled between London and Niagara Falls (in either direction)?



(A) 35 (**B**$*$) 27 (C) 48 (D) 21 (E) 30

Solution

Let

- $TL =$ the number of outgoing trains from Toronto to London
- $TN =$ the number of outgoing trains from Toronto to Niagara Falls
- $LN =$ the number of outgoing trains from London to Niagara Falls
- $LT =$ the number of outgoing trains from London to Toronto
- $NL =$ the number of outgoing trains from Niagara Falls to London
- $NT =$ the number of outgoing trains from Niagara Falls to Toronto

Then

$$\begin{cases} TL + NL + LN + LT = 190 \\ TN + LN + NL + NT = 72 \\ TL + TN + LT + NT = 208 \end{cases}$$

Note that $NL + LN$ equals the desired number of trains traveled between London and Niagara Falls in either direction.

Adding the first equation to the second and subtracting the third from the result we obtain

$$NL + LN = \frac{1}{2}(190 + 72 - 208) = 27.$$

Answer: 27.

Answer: B

10. Find $\sin 30^\circ + \sin 300^\circ + \sin 3000^\circ + \cdots + \sin\left(3 \times 10^{1000}\right)^\circ$.

(**A**∗) $\frac{1}{2} + 997\frac{\sqrt{3}}{2}$ (B) 0 (C) $\frac{1}{2} - \frac{\sqrt{3}}{2}$ (D) 500 (E) $225(1+\sqrt{3})$

Solution

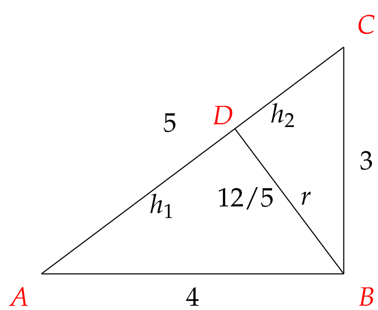
$$\frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \cdots + \frac{\sqrt{3}}{2} = \frac{1}{2} + 997\frac{\sqrt{3}}{2}$$

Answer: A

11. A right triangle with legs 3 cm and 4 cm is rotated around its hypotenuse. Determine the volume of the formed solid of revolution.

(A) 5π cm^3 (B) $\frac{16\pi}{3}$ cm^3 (C) $\frac{64\pi}{9}$ cm^3 (D) 12π cm^3 (**E**∗) $\frac{48\pi}{5}$ cm^3

Solution



Let the triangle be denoted by ABC with the hypotenuse AC and legs AB of length 4 and BC of length 3. Let BD be the altitude from B. Rotating the triangle about the hypotenuse AC we see that the formed solid of revolution will consist of two circular cones each of radius $BD = r$ and of heights $DC = h_1$ and $DA = h_2$, respectively. The volume of the solid now becomes

$$\frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = \frac{1}{3}\pi r^2 (h_1 + h_2) = \frac{1}{3}\pi r^2 AC$$

where $AC = 5$. All we need is to find the common radius r of the two cones. From the similar triangles ABC and BDC, $\frac{BC}{BD} = \frac{AC}{AB}$ or $\frac{3}{r} = \frac{5}{4}$ and so $r = \frac{12}{5}$. The volume of the solid equals

$$\frac{1}{3}\pi \frac{12^2}{5^2} 5 = \frac{48\pi}{5}$$

Answer: $\frac{48\pi}{5}$ cm^3.

Answer: E

12. In a town, there are only 2 types of residents: Those who only tell the truth and those who only tell lies. 150 residents fill out a questionnaire. Below are the results of the questionnaire. How many residents always lie?

Question	Number of answers "Yes"
Are you under 18 years old?	68
Are you from 18 to 35 years old?	45
Are you from 36 to 65 years old?	52
Are you over 65 years old?	75
Do you have kids?	72

(A) 23 (B) 30 (**C**∗) 45 (D) 75 (E) 86

Solution

Only consider the age questions. The total number of answers "Yes" equals $68 + 45 + 52 + 75 = 240$. Let T be the number of residents who tell the truth and let L be the number of residents who tell lies. Note that in the age questions each resident who tells lies, answered "Yes" 3 times. We obtain

$$\begin{cases} T + 3L = 240 \\ T + L = 150 \end{cases} \implies L = 45.$$

$\boxed{\textbf{Answer: C}}$