BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2026

Senior Final, Part A Problems & Solutions

1. If f is a function such that $f(0) = 1$, $f(1) = 3$ and

$$f(n+2) = 2f(n) - f(n+1),$$

then $f(5)$ equals

(A) -7 (B) -3 (C) 7 (D) 10 (**E**∗) 23

Solution

For $n = 0$, $f(2) = 2f(0) - f(1) = 2 \cdot 1 - 3 = -1$.
For $n = 1$, $f(3) = 2f(1) - f(2) = 2 \cdot 3 + 1 = 7$.
For $n = 2$, $f(4) = 2f(2) - f(3) = 2 \cdot (-1) - 7 = -9$.
For $n = 3$, $f(5) = 2f(3) - f(4) = 2 \cdot 7 + 9 = 23$.

Answer: E

2. Suzanne and her father share a birthday. This year, they notice that if they reverse the digits of Suzanne's age, they get her father's age (and vice versa). Assuming they are both under 90 years of age, what is the minimum number of years that must pass before this can happen again?

(A) 10 (**B**∗) 11 (C) 15 (D) 22 (E) 30

Solution

Suppose Suzanne's father's age is $10a + b$. Then Suzanne's age is $10b + a$. Their age difference is then

$$10a + b - 10b - a = 9a - 9b = 9(a - b).$$

Since a and b are digits, the age difference is a multiple of 9 (but 0, 9, 72 and 81 would be impossible). Therefore, $a - b$ is 2, 3, 4, 5, 6 or 7.

If $a - b = 2$, they are 18 years apart, and their digits will reverse at

$$(13, 31),\ (24, 42),\ (35, 53),\ (46, 64),\ (57, 75),\ (68, 86) \text{ and } (79, 97).$$

If $a - b = 3$, they are 27 years apart, and their digits will reverse at

$$(14, 41),\ (25, 52),\ (36, 63),\ (47, 74),\ (58, 85),\ \text{and } (69, 96).$$

If $a - b = 4$, they are 36 years apart, and their digits will reverse at

$$(15, 51),\ (26, 62),\ (37, 73),\ (48, 84),\ \text{and } (59, 95).$$

If $a - b = 4$, they are 45 years apart, and their digits will reverse at

$$(16, 61),\ (27, 72),\ (38, 83),\ \text{and } (49, 94).$$

If $a - b = 6$, they are 54 years apart, and their digits will reverse at

$$(17, 71),\ (28, 82),\ \text{and } (39, 93).$$

If they $a - b = 7$, they are 63 years apart, and their digits will reverse at

$$(18, 81), \text{ and } (29, 92).$$

Note that since their age difference $a - b$ remains constant, a and b increase at the same rate, so if b goes up by one, a must as well. If they are both under 90 and their ages are reversed, it will happen again in 11 years at the earliest.

Answer: B

3. For how many integers n is the number $30n^4 + 5n^2 + 10$ a perfect square?

(**A**∗) 0 (B) 1 (C) 2 (D) 4 (E) more than 4

Solution

The equation

$$5(6n^4 + n^2 + 2) = m^2$$

implies that m is a power of 5. However, $6n^4 + n^2 + 2$ is not divisible by 5 for any n because n^4 gives the remainder of 0 or 1 on division by 5 and n^2 gives the remainder of 0, 1 or 4 on division by 5. Answer: There is no such n.

Answer: A

4. How many solution pairs (x, y) does the following system of equations have? Note: x and y can be any real numbers.

$$\begin{cases} y^2 = 25 - x^2 \\ y + x = x^2 - 6 \end{cases}$$

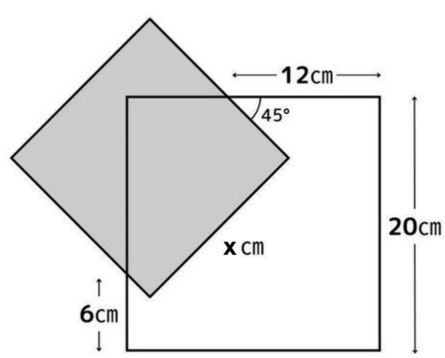
(A) 0 (B) 1 (C) 2 (D) 3 (**E**∗) 4

Solution

The first curve $x^2 + y^2 = 25$ is a circle of radius 5 centered at the origin. The second curve $y = x^2 - x - 6$ is a parabola with the vertex $(0.5, -6.25)$ and its branches up. Note, the vertex of the parabola is entirely below the circle in the plane. So, there are four distinct intersection points of the two curves.

Answer: E

5. Consider two squares, one larger square of side length 20 cm, and one smaller square (shaded) of side length x cm. From the information given, find the area of the smaller square.



(A) 256cm^2 (B) 300cm^2 (C) 224cm^2 (**D**$*$) 242cm^2 (E) 275cm^2

Solution

Translate the shaded square Southeast keeping the given 45° angle until the bottom-right side of it is contained in the diagonal of the large square. Then, the side length of the shaded square equals

$$\sqrt{800} - 6\sqrt{2} - 3\sqrt{2} = 20\sqrt{2} - 6\sqrt{2} - 3\sqrt{2} = 11\sqrt{2} \text{ cm.}$$

The area of the shaded square is then $11^2 \times 2 = 242\text{cm}^2$.

Answer: D

6. Calculate the following:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{2025}+\sqrt{2026}}$$

(A) $\sqrt{2025}+1$ (**B**$*$) $\sqrt{2026}-1$ (C) 2026 (D) $\sqrt{2026}$ (E) $\sqrt{2026}+2025$

Solution

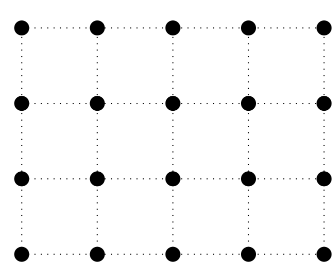
Use the conjugates:

$$\frac{1}{\sqrt{1}+\sqrt{2}} \cdot \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} + \frac{1}{\sqrt{2}+\sqrt{3}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \cdots + \frac{1}{\sqrt{2025}+\sqrt{2026}} \cdot \frac{\sqrt{2026}-\sqrt{2025}}{\sqrt{2026}-\sqrt{2025}}$$

$$\frac{\sqrt{2}-\sqrt{1}}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \cdots + \frac{\sqrt{2026}-\sqrt{2025}}{2026-2025} = \sqrt{2026}-1.$$

Answer: B

7. A rectangular grid of 20 dots contains the corners of twelve 1×1 squares, six 2×2 squares, and two 3×3 squares, for a total of 20 squares with sides parallel to the sides of the grid. Find the smallest number of dots that can be removed so that each of the 20 squares is missing at least one corner.

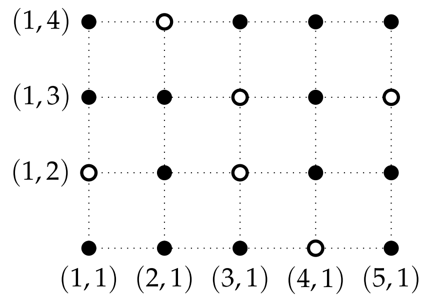


(A) 4 (B) 5 (**C**$*$) 6 (D) 7 (E) 8

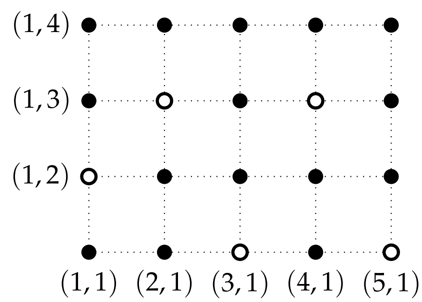
Solution

Assume that the grid is placed in a plane coordinate system with integer coordinates of $x = 1, 2, 3, 4, 5$ and $y = 1, 2, 3, 4$. To find the smallest number m of dots one can remove instead find the largest number

M of dots one can keep to ensure that no square has all 4 corners present. There are 5 columns and each column has 4 points. A square needs two columns that both contain the same two y-values. So to avoid squares we make column patterns incompatible with each other. We proceed from the left to the right removing the small squares first. For example, in column 1, keep 3 dots and remove the point $(1, 2)$. In column 2 we cannot keep all the four dots, but we can keep three by removing the point $(2, 1)$. In column 3 we must remove two points $(3, 2)$ and $(3, 3)$ to avoid 1×1 squares there. In column 4 one can remove only one point, say $(4, 1)$. Finally, in column 1 we remove one point $(5, 3)$. The number of points left is 14 and we removed 6 points. One can check that each of the 20 squares is missing at least one corner (refer to the diagram below).



Now show that we can't improve by keeping 15 or more points. To see this, by the contrary, assume we kept 15 points (see the diagram below). We were able to avoid all the 1×1 squares by removing exactly one dot in each of the 5 columns (otherwise we would have a 1×1 square). However, this always forces at least one 2×2 square with all the corners present.



So, with keeping 15 or more points, there is too much overlap between columns. No matter how we arrange them at least one square stays. So the smallest number of dots one has to remove is 6.

Answer: C

8. A pile of \$1 coins is to be divided among three children. The first child is entitled to $\frac{1}{2}$ of the coins, the second to $\frac{1}{3}$ of the coins, the third to $\frac{1}{6}$ of the coins. Each child takes some number of coins from the pile so that no coins remain. The first child then returns $\frac{1}{2}$ of their coins, the second returns $\frac{1}{3}$ of their coins and the third returns $\frac{1}{6}$ of their coins. The returned coins are combined and divided equally among the three children. After this process, each child has exactly the number of coins to which they are entitled. What is the smallest possible number of coins in the pile?

(A) 102 (B) 186 (**C**$*$) 282 (D) 324 (E) 360

Solution

Let \$$x$ be the total amount of money and assume that children A, B and C have their shares as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$, respectively. Denote by a and b the amounts that child A and child B took from the pile, respectively.

Then child C took $x - a - b$. The total amount that the three children later returned to the pile equals

$$\frac{1}{2}a + \frac{1}{3}b + \frac{1}{6}(x - a - b) = \frac{1}{3}a + \frac{1}{6}b + \frac{1}{6}x.$$

By the conditions, the above amount divided by 3 given back to each of the children results in their original portions they are entitled to. This is expressed in the following system of equations.

$$\begin{cases} \frac{1}{2}a + \frac{1}{9}a + \frac{1}{18}b + \frac{1}{18}x = \frac{1}{2}x \\ \frac{2}{3}b + \frac{1}{9}a + \frac{1}{18}b + \frac{1}{18}x = \frac{1}{3}x \\ \frac{5}{6}(x - a - b) + \frac{1}{9}a + \frac{1}{18}b + \frac{1}{18}x = \frac{1}{6}x \end{cases}$$

Multiplying each equation by 18 and simplifying we obtain

$$\begin{cases} 11a + b = 8x \\ 2a + 13b = 5x \\ 13a + 14b = 13x \end{cases}$$

Note that the third equation equals the sum of the first two. So, the third equation is not required.

$$\begin{cases} 11a + b = 8x \\ 2a + 13b = 5x \end{cases}$$

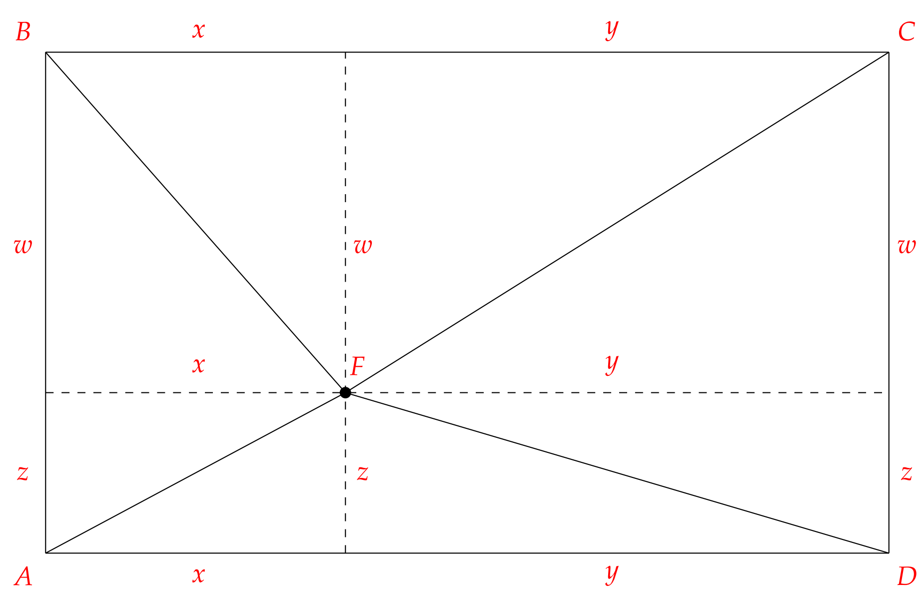
Eliminating first b and then a we find that $141a = 99x$ and $141b = 39x$. Therefore, $\frac{a}{b} = \frac{99}{39}$ and since 39 is a prime, the lowest possible integers are $a = 99$ and $b = 39$. This implies $x = 141$. However, 141 is not divisible by 2 which means that x must be increased. Double each of a, b and x. Then, $a = 198$, $b = 78$ and $x = 282$. Note that this time $x = 282$ is divisible by 6 which gives the smallest possible total integer amount. We also obtain that child C took $c = x - a - b = 282 - 198 - 78 = 6$ from the pile. Answer: 282.

$\boxed{\textbf{Answer: C}}$

9. A fly is sitting on the ceiling of a rectangular room. The distances from the fly to the three closest corners on the floor are 3m, 4m and 5m. Find the distance from the fly to the fourth corner on the floor.

(A) 8 (B) $2\sqrt{10}$ (C) 6 (D) $5\sqrt{3}$ (**E**$*$) $4\sqrt{2}$

Solution



The diagram shows the floor of the room where the point F is the vertical projection of the position of the fly from the ceiling to the floor. Assume that the height of the room is h and denote by x, y, z and w the distances shown in the picture. Without loss of generality, in this diagram, the distance from the fly to the bottom corners A, B and D, are 3, 4 and 5m, respectively. We need to find the distance from the fly to the bottom corner C. By the Theorem of Pythagoras,

$$\begin{cases} x^2+z^2=9-h^2 \\ x^2+w^2=16-h^2 \\ y^2+z^2=25-h^2 \end{cases}$$

From the second and third equations,

$$\begin{aligned} y^2+w^2 &= 25-h^2-z^2+16-h^2-x^2 \\ &= 41-h^2-(h^2+x^2+z^2) \end{aligned}$$

By the first equation,

$$y^2+w^2=41-h^2-9=32-h^2$$

So, the distance from the fly to the fourth corner is $\sqrt{y^2+w^2+h^2}=\sqrt{32}=4\sqrt{2}$.

Answer: E

10. Two hikers start at the same time. The first hiker walks from point A to point B, the second from B to A. Each hiker walks at a constant speed. When the first hiker is halfway to B, the second hiker has 24 km remaining. When the second hiker is halfway to A, the first hiker has 15 km remaining. Find the distance between A and B.

(A) 36 (**B**$*$) 40 (C) 32 (D) 48 (E) 12

Solution

Let d be the distance between A and B and let v_1 and v_2 be the speeds of the first and the second hiker, respectively. Then

$$\begin{cases} \dfrac{d}{2v_1}=\dfrac{d-24}{v_2} \\[2ex] \dfrac{d-15}{v_1}=\dfrac{d}{2v_2} \end{cases} \implies \frac{2d-48}{d}=\frac{d}{2(d-15)}$$

Solving the corresponding quadratic equation we obtain $d=12$ or $d=40$. Answer: $d=40$.

Answer: B