BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2026

Junior Preliminary Problems & Solutions

1. Given $2^x = 8^{y+1}$ and $9^y = 3^{x-9}$ the value of $x + y$ is:

(A) 18 (B) 21 (C) 24 (**D**$*$) 27 (E) 33

Solution

$$\begin{cases} 2^x = 8^{y+1} \\ 9^y = 3^{x-9} \end{cases} \implies \begin{cases} 2^x = 2^{3y+3} \\ 3^{2y} = 3^{x-9} \end{cases} \implies \begin{cases} x = 3y + 3 \\ 2y = x - 9 \end{cases}$$

$$2y = 3y + 3 - 9 \implies y = 6, \quad x = 21$$

Therefore, $x + y = 21 + 6 = 27$.

Answer: D

2. The factors of 6 are 1, 2, 3, 6, and the sum of all factors is 12. What is the sum of all the factors of 2026?

(A) 2027 (**B**$*$) 3042 (C) 4149 (D) 6084 (E) 7156

Solution

The factors of 2026 are 1, 2, 1013, 2026. The sum of all factors equals 3042.

Answer: B

3. In a quadrilateral the four sides and one of the diagonals were measured. The obtained lengths are 1, 2, 2.8, 5 and 7.5, but not in any order. What is the length of the measured diagonal? (Recall that a quadrilateral is a four-sided polygon.)

(A) 1 (B) 2 (**C**$*$) 2.8 (D) 5 (E) 7.5

Solution

The quadrilateral is broken into two triangles by the given measured diagonal. The measured diagonal of the quadrilateral will be a side of each of the two triangles and so the triangle inequality must be satisfied for both triangles. Each row in the following table has two triples of side lengths where the first value in each triple is the possible length of the measured diagonal. Check where two valid triangles are formed.

Triangle inequality holds	Diagonal			Diagonal			Triangle inequality holds
yes $\rightarrow$	1	2	2.8	1	5	7.5	$\leftarrow$ no
no $\rightarrow$	1	2	5	1	2.8	7.5	$\leftarrow$ no
no $\rightarrow$	1	2	7.5	1	2.8	5	$\leftarrow$ no
no $\rightarrow$	2	1	2.8	2	5	7.5	$\leftarrow$ no
no $\rightarrow$	2	1	5	2	2.8	7.5	$\leftarrow$ no
no $\rightarrow$	2	1	7.5	2	2.8	5	$\leftarrow$ no
yes $\rightarrow$	2.8	1	2	2.8	5	7	$\leftarrow$ yes
no $\rightarrow$	5	1	2	5	2.8	7.5	$\leftarrow$ no
no $\rightarrow$	5	1	2.8	5	2	7.5	$\leftarrow$ no
no $\rightarrow$	5	1	7.5	5	2	2.8	$\leftarrow$ no
no $\rightarrow$	7.5	1	2	7.5	2.8	5	$\leftarrow$ no
no $\rightarrow$	7.5	1	2.8	7.5	2	5	$\leftarrow$ no
no $\rightarrow$	7.5	1	5	7.5	2	2.8	$\leftarrow$ no

Only when the measured diagonal has length 2.8, we obtain two valid triangles. The answer is 2.8.

Answer: C

4. In five years from now Michael's age related to Erin's age will be 8 : 7. Find the sum of Michael's and Erin's ages now, given that one year ago Michael was twice as old as Erin.

(**A**∗) 5 (B) 9 (C) 14 (D) 18 (E) 26

Solution

Let x be Michael's age and let y be Erin's age. We obtain:

$$\begin{cases} 7(x+5) = 8(y+5) \\ x-1 = 2(y-1) \end{cases} \implies \begin{cases} 7x - 8y = 5 \\ x = 2y - 1 \end{cases} \implies x = 3, \quad y = 2 \implies x + y = 5$$

Answer: A

5. In a triangle ABC the degree measures of the angles A and B equal 50° and 70°, respectively. The two altitudes from the vertices A and B intersect at the point O. Find the degree measure of the angle AOB.

(A) 60° (B) 90° (C) 110° (**D**∗) 120° (E) 135°

Solution

First note that $\angle C = 180^\circ - 50^\circ - 70^\circ = 60^\circ$. Let D be the foot of the altitude from A and let E be the foot of the altitude from B. The sum of the angles of the quadrilateral $ODCE$ equals $\angle DOE + 90^\circ + 90^\circ + 60^\circ = 360^\circ$. So $\angle DOE = 120^\circ$ which implies $\angle AOB = 120^\circ$ by vertical angles.

Answer: D

6. How many two-digit numbers satisfy each of the three following statements?

- This number ends with 5 or it is divisible by 7 (or both).
- This number is greater than 20 or it ends with 9 (or both).
- This number is divisible by 12 or it is less than 21 (or both).

(A) 0 (**B**∗) 1 (C) 2 or 3 (D) 4 or 5 (E) more than 5

Solution

For each of the three conditions list all the numbers that satisfy it. We must find numbers that satisfy all the three conditions.

- 15, 25, 35, 45, 55, 65, 75, 85, 95 or 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98
- 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 or 19, 29, 39, 49, 59, 69, 79, 89, 99

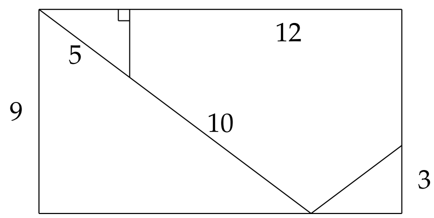
- $\boxed{12, 24, 36, 48, 60, 72, 84, 96}$ or $\boxed{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}$

  If the given number is less than 21 then it must end with 9 and so it must be also divisible by 7. No such number exists.

  If the given number is greater than 20, then it must be divisible by 12 and therefore can't end with 5. So it must be divisible by 7 as well. Only 84 works and so, only one solution.

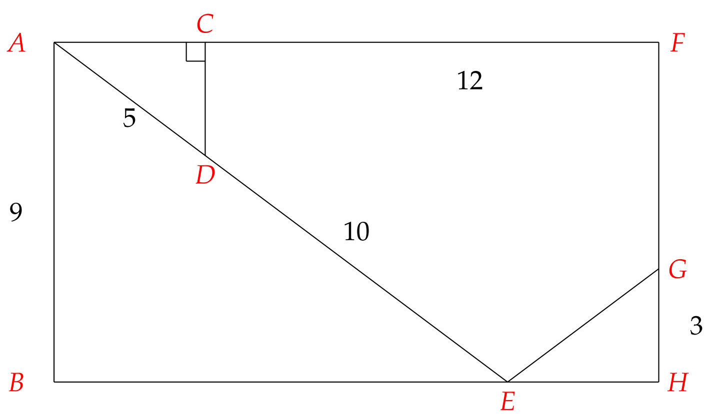
Answer: B

7. A rectangle is cut into pieces as shown. If the pieces are rearranged to form a square, then what is the perimeter of the square?



(**A***) 48 (B) 64 (C) $16\sqrt{3}$ (D) 144 (E) $24\sqrt{2}$

Solution



From the right triangle ABE, $BE = \sqrt{AE^2 - AB^2} = \sqrt{15^2 - 9^2} = 12$. The right triangles ABE and DCA are similar with the scaling factor of $1/3$. So, $CD = 3$ and $AC = 4$. We obtain $AF = BH = 16$ and this implies that $EH = BH - BE = 16 - 12 = 4$. We see that the right triangles DCA and GHE are congruent. By shifting the triangle ABE Southeast along the line AE until A coincides with the original position of D and putting the triangle DCE in the bottom right corner of the resulting figure, we obtain a square of side 12. Its perimeter is $P = 48$.

Answer: A

8. Five people are accused of a robbery, but only one is guilty. Each gives a statement, but exactly two of the statements are false. This is enough to give a decisive answer. What single person is guilty?

Alex: It was Ben.

Ben: Alex is lying.

Cindy: Dennis is innocent.

Dennis: Ellie is innocent.

Ellie: Dennis is telling the truth.

(A) Alex (B) Ben (C) Cindy (**D**$*$) Dennis (E) Ellie

Solution

Denote the statements given by Alex, Ben, Cindy, Dennis and Ellie by A, B, C, D and E, respectively. The following table shows the truth values of these five statements corresponding to each of the five possibilities when exactly one person is guilty.

	A	B	C	D	E
Alex is guilty	False	True	True	True	True
Ben is guilty	True	False	True	True	True
Cindy is guilty	False	True	True	True	True
Dennis is guilty	False	True	False	True	True
Ellie is guilty	False	True	True	False	False

Exactly two false statements appear only when Alex and Cindy are lying. So, Dennis is guilty.

Answer: D

9. There are 200 units in a new condo tower in Metro Vancouver. In every condo, either one person lives alone, or 2 or 3 may share. The total number of people living in units 1 to 102 is 106, and the total number of people living in units 101 to 200 is 300. How many people live in this new condo tower?

(A) 398 (**B**$*$) 400 (C) 397 (D) 403 (E) 406

Solution

1 person in each unit from 1 to 100 and 3 persons in each unit from 101 to 200. The answer is 400 in total.

Answer: B

10. The 6-digit number $A2026A$ is a multiple of 8. The 6-digit number $B2026A$ is a multiple of 11. Find the value of $A + B$.

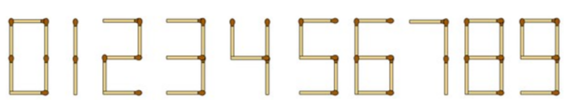
(A) 9 (**B**$*$) 6 (C) 5 (D) 11 (E) 8

Solution

Since $A2026A$ is a multiple of 8, the integer formed from its three last digits, namely $26A$, is divisible by 8. So, $A = 4$. Since $B2026A$ is a multiple of 11, the alternating sum of its digits must me divisible by 11. So, $B - 2 + 0 - 2 + 6 - A = B - 2 + 0 - 2 + 6 - 4 = B - 2$ must be divisible by 11. This implies that $B = 2$. Therefore, $A + B = 4 + 2 = 6$.

Answer: B

11. Matchsticks can form each of the ten digits as follows:



How many 3-digit odd numbers can be formed using an odd number of matchsticks? For example

is a 3-digit odd number formed using an odd number of matchsticks.

(A) 250 (B) 185 (C) 148 (D) 320 (**E**∗) 225

Solution

Digit	# of sticks
0	6
1	2
2	5
3	5
4	4
5	5
6	6
7	3
8	7
9	6

For an odd three-digit number the first digit cannot be 0 and the last digit must be odd.

In order to use an odd number of sticks we must have either all the three digits with an odd number of sticks each or exactly two digits with an even number of sticks and one with an odd number of sticks.

Denote by E, respectively, O a digit formed with an even, respectively, odd number of matchsticks.

Then $E:$ 0, 1, 4, 6, 9 and $O:$ 2, 3, 5, 7, 8.

We obtain the following possibilities:

Number	# of ways
EEO	$4 \cdot 5 \cdot 3 = 60$
EOE	$4 \cdot 5 \cdot 2 = 40$
OEE	$5 \cdot 5 \cdot 2 = 50$
EEO	$5 \cdot 5 \cdot 3 = 75$

So, there are $60 + 40 + 50 + 75 = 225$ of such numbers in total.

Answer: E

12. In a town, there are only 2 types of residents: Those who only tell the truth and those who only tell lies. 150 residents fill out a survey and below are the results. How many residents always lie?

Question	Number of answers "Yes"
Are you under 18 years old?	68
Are you from 18 to 35 years old?	45
Are you from 36 to 65 years old?	52
Are you over 65 years old?	75
Do you have kids?	72

(A) 23 (B) 30 (**C**∗) 45 (D) 75 (E) 86

Solution

Only consider the age questions. The total number of answers "Yes" equals $68 + 45 + 52 + 75 = 240$. Let T be the number of residents who tell the truth and let L be the number of residents who tell lies. Note that in the age questions each resident who tells lies, answered "Yes" 3 times. We obtain

$$\begin{cases} T + 3L = 240 \\ T + L = 150 \end{cases} \implies L = 45.$$

$\boxed{\textbf{Answer: C}}$