BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2026

Junior Final, Part B Problems & Solutions

1. Find all triples of primes a, b and c such that $7a - bc = 105$.

   Solution

   Rewriting as $bc = 7a - 105 = 7(a - 15)$ we see that bc must be divisible by 7. Since both b and c are primes one of them must equal 7. Consider the following cases.

   Case 1. $b = 7$.

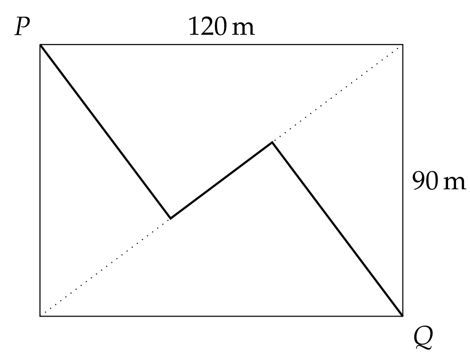
   Then $7c = 7(a - 15)$ and so $c = a - 15$. Since both a and c are primes with $a > 15$, we see that a must be odd. So $c = a - 15$ must be even which forces $c = 2$, the only even prime possible. Then $a = 17$ which is also a prime. So, one possible solution is $(a, b, c) = (17, 7, 2)$.

   Case 2. $c = 7$.

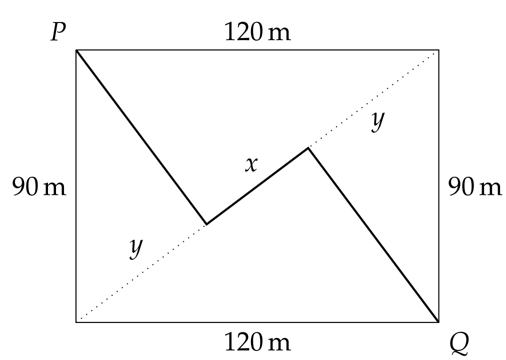
   Similarly as in Case 1, the only possibility is $b = 2$ and $a = 17$. So, another solution is the triple $(a, b, c) = (17, 2, 7)$.

   We conclude that the only triples of primes (a, b, c) satisfying the given equation are $(17, 7, 2)$ and $(17, 2, 7)$.

2. Anne of Green Gables is at the corner entrance of the "Park of the Lake of Shining Waters." Starting at point P, she needs to reach point Q as quickly as possible. If she walks along the sides of the park, she walks a total distance of 210 metres. If she walks through the park, she must follow the zigzag path with two right-angled turns as shown. Is this zigzag path shorter than 210 metres?



   Solution



The length of the diagonal of the rectangle equals $\sqrt{120^2+90^2} = 150$ by the Theorem of Pythagoras. Let us introduce unknown distances x and y as in the diagram. We obtain the following system of equations:

$$\begin{cases} x+2y = 150 \\ 90^2 - y^2 = 120^2 - (x+y)^2 \end{cases} \qquad \begin{cases} y = 75 - x/2 \\ x^2 + 2xy = 6300 \end{cases} \qquad \begin{cases} y = 75 - x/2 \\ x^2 + 2x(75 - x/2) = 6300 \end{cases} \qquad \begin{cases} y = 54 \\ x = 42 \end{cases}$$

Then the length of the zigzag equals

$$2\sqrt{90^2 - y^2} + x = 2\sqrt{90^2 - 54^2} + 42 = 2 \cdot 72 + 42 = 186 \text{ m}.$$

We conclude that the length of the zigzag is shorter than 210 metres.

3. You have 100 large jugs, each with a capacity of 10 Litres. For each integer $n = 1, \ldots, 100$, there is a jug that contains exactly n mL of water. You are allowed to redistribute the water among the jugs by pouring water from one jug into another as many times as you like subject to the rule that, at each step in the process, the amount of water poured into a jug must equal the amount that is in the jug already (so that the effect of the added water is to double the amount of water in the jug).

   a) Is it possible to redistribute the water so that, for each $n = 1, \ldots, 100$, there is a jug that contains exactly n mL of water, but no jug contains the same amount of water as it did initially? Explain.

   b) Is it possible to redistribute the water so that five of the jugs each contain exactly 3 mL of water and, for each $n = 6, 7, \ldots, 100$, there is a jug that contains exactly n mL of water? Explain.

Solution

a) Yes. Proceed as follows

$$100 \longrightarrow 1, \quad 99 \longrightarrow 2, \quad 98 \longrightarrow 3, \quad \ldots, \quad 51 \longrightarrow 50$$

by pouring from jug A to jug B the amount that jug B initially has. The result is

$$2,\ 4,\ 6,\ \ldots,\ 100,\ 1,\ 3,\ 5,\ \ldots\ ,\ 97,\ 99.$$

b) No. Following the given rule, the quantity of jugs containing odd numbers of millilitres of water should never increase.

4. Lewis and Carol travel together on a road from A to B, then return on the same road, with the entire trip taking 3 hours. Sometimes the road goes uphill, sometimes it goes downhill, and sometimes it is level. When the road goes uphill their rate is 40 kph; when it goes downhill their rate is 60 kph; on level road their rate is x kph.

   Even if you were given a numerical value for x, the distance from A to B would (in most cases) not be uniquely determined. But there is one value for x that would determine that distance uniquely. What is this value? Note: Uphill is going downhill returning!.

Solution

Let d_1, d_2 and d_3 be the distances of the uphill, downhill, and flat portions of the journey from A to B, respectively, so that we may summarize the given information in the table below.

	speed	distance
uphill	40kph	d_1
downhill	60kph	d_2
flat	x	d_3

Using the formula $v = d/t$, we obtain

$$3 = \left(\frac{d_1}{40} + \frac{d_2}{60} + \frac{d_3}{x}\right) + \left(\frac{d_1}{60} + \frac{d_2}{40} + \frac{d_3}{x}\right),$$

or

$$3 = \frac{d_1 + d_2}{40} + \frac{d_1 + d_2}{60} + \frac{2d_3}{x},$$

which yields

$$360x = 5x(d_1 + d_2) + 240d_3.$$

Note that if $5x = 240$, then we arrive at

$$360x = 240(d_1 + d_2 + d_3),$$

which uniquely determines $d_1 + d_2 + d_3 = \dfrac{360x}{240}$ to be the distance from A to B. Hence this distance is uniquely determined if $5x = 240$, which forces $x = 48$.

5. Let us call a positive integer "nice" if it can be expressed as the sum of a three-digit number abc and a three-digit number cba, where the symbols a, b and c represent digits and the digits a and c are not zero. How many positive integers are nice?

Solution

For digits a, b and c with $a \neq 0$ and $c \neq 0$ consider the column addition of the numbers abc and cba.

$$\begin{array}{rccc} & a & b & c \\ + & c & b & a \\ \hline & \square & \square & \square \end{array} \qquad \text{or} \qquad \begin{array}{rcccc} & & a & b & c \\ + & & c & b & a \\ \hline & \square & \square & \square & \square \end{array}$$

We will count the number of different three-digit and four-digit nice numbers in the following cases.

Case 1. $2 \le a + c \le 9$ and $0 \le b \le 4$.

Then the sum is three-digit. Below are the possibilities for the digits:

$a+c$	$2b$	$c+a$
2, 3, 4, 5, 6, 7, 8, 9	0, 2, 4, 6 , 8	2, 3, 4, 5, 6, 7, 8, 9

So, $8 \times 5 = 40$ different numbers.

Case 2. $2 \le a + c \le 9$ and $5 \le b \le 9$. Then $2b \ge 10$ and we carry 1 over.

Then the sum is three or four-digit. Below are the possibilities for the digits:

When $a + c < 9$ the nice number is still three-digit.

$a+c+1$	$2b-10$	$c+a$
3, 4, 5, 6, 7, 8, 9	0, 2, 4, 6 , 8	2, 3, 4, 5, 6, 7, 8

When $a + c = 9$, the nice number is four-digit with 1 for the thousands and 0 for the hundredths.

	$a+c+1-10$	$2b-10$	$c+a$
1	0	0, 2, 4, 6 , 8	9

So, $7 \times 5 + 5 = 40$ different numbers.

Case 3. $10 \leq a + c \leq 18$ and $0 \leq b \leq 4$.

Then the nice number is four-digit. Below are the possibilities for the digits:

	$a + c - 10$	$2b + 1$	$c + a - 10$
1	0, 1, 2, 3, 4, 5, 6, 7, 8	1, 3, 5, 7 , 9	0, 1, 2, 3, 4, 5, 6, 7, 8

So, $9 \times 5 = 45$ different numbers.

Case 4. $10 \leq a + c \leq 18$ and $5 \leq b \leq 9$.

Then the nice number is four-digit. Below are the possibilities for the digits:

	$a + c + 1 - 10$	$2b + 1 - 10$	$c + a - 10$
1	1, 2, 3, 4, 5, 6, 7, 8, 9	1, 3, 5, 7 , 9	0, 1, 2, 3, 4, 5, 6, 7, 8

So, $9 \times 5 = 45$ different numbers.

In total we obtain $40 + 40 + 45 + 45 = 170$ different nice numbers.