BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2026

Junior Final, Part A Problems & Solutions

1. A book has n pages. If 1392 digits are needed to number the pages from 1 to n, how many pages are in the book?

   (A) 497 (B) 498 (C) 499 (**D**$*$) 500 (E) 501

   Solution

   $9 \cdot 1$ digits are required to enumerate the first 9 pages of the book.

   $(99 - 10 + 1) \cdot 2 = 180$ digits are required to enumerate pages 10 to 99.

   $(999 - 100 + 1) \cdot 3 = 2700$ digits are required to enumerate pages 100 to 999.

   Since 1392 digits must be used, the number of pages of the book is greater that 99 but less than 999. Moreover, there are exactly $1392 - 9 \cdot 1 - 90 \cdot 2 = 1203$ digits to be used for the 3-digit numbers in the enumeration. So, the number of 3-digit pages equals $1203/3 = 401$. Therefore, the book has $9 + 90 + 401 = 500$ pages.

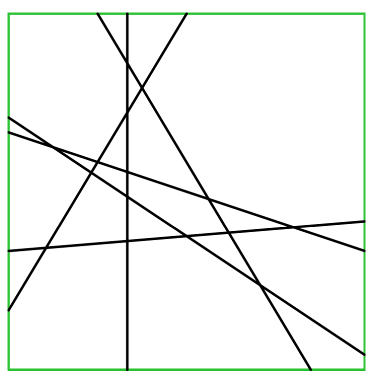
   Answer: D

2. Six straight lines are drawn in the plane with no two being parallel and no three intersecting at the same point. The number of regions into which the plane is divided is:

   (A) 16 (B) 20 (**C**$*$) 22 (D) 24 (E) 26

   Solution

   Since no two lines are parallel and none three of them intersect at the same point, each line will have a separate point of intersection with every other line. In order to practically be able to count the regions, the diagram should be drawn so that all of these points of intersection are visible. When this is done, it can be seen that the plane is divided into 22 regions.

   Answer: C

3. As a number of passengers enter an empty streetcar, exactly 2/7 of them took seats. At the next stop, the number of passengers increased exactly by 15%. Find the number of passengers who entered at the

start, given that the streetcar can hold at most 180 passengers.

(A) 70 (B) 133 (**C**$*$) 140 (D) 154 (E) 168

Solution

The initial number of passengers is divisible by 7 and 20 (Note that 15% is $3/20$ of the whole). So, the initial number is divisible by 140, but it was smaller than 180. So, the initial number is 140.

$\boxed{\textbf{Answer: C}}$

4. Suzanne and her father share a birthday. This year, they notice that if they reverse the digits of Suzanne's age, they get her father's age (and vice versa). Assuming they are both under 90 years of age, what is one of the possible age differences between Suzanne and her father?

(A) 24 (**B**$*$) 27 (C) 29 (D) 33 (E) 38

Solution

Suppose Suzanne's father's age is $10a + b$. Then Suzanne's age is $10b + a$. Their age difference is then

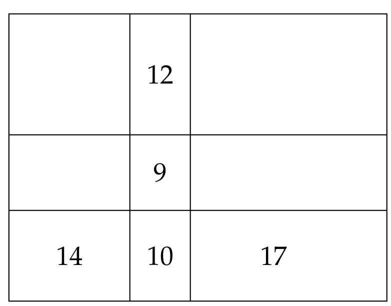
$$10a + b - 10b - a = 9a - 9b = 9(a - b).$$

Since a and b are digits, the age difference is a multiple of 9 (but 0, 9, 72 and 81 would be impossible). Therefore, $a - b$ is 2, 3, 4, 5, 6 or 7.

So, they can be 18, 27, 36, 45, 54 or 63 years apart. Only 27 is among the given choices.

$\boxed{\textbf{Answer: B}}$

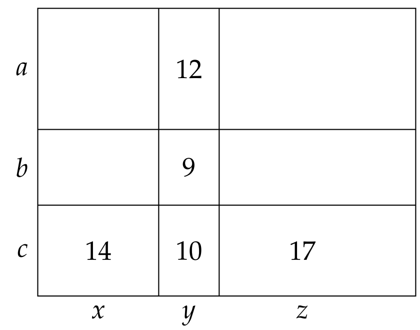
5. A rectangle is subdivided into nine smaller non-overlapping rectangles by two pairs of parallel lines. The number inside a small rectangle denotes its perimeter. Find the perimeter of the larger rectangle.



(**A**$*$) 42 (B) 44 (C) 48 (D) 50 (E) 54

Solution

Denote by a, b, c, x, y and z the unknown lengths as in the diagram below.



Then

$$\begin{cases} a+b+c+3y = \frac{1}{2}(12+9+10) = 15.5 \\ x+y+z+3c = \frac{1}{2}(14+10+17) = 20.5 \end{cases}$$

$$a+b+c+x+y+z = 15.5+20.5-3(y+c) = 36-3(10\cdot 0.5) = 21.$$

Therefore, the perimeter of the larger rectangle is $2\cdot 21 = 42$.

Answer: A

6. In a chess tournament every competitor played exactly one game against each other opponent. Each competitor scored 1 point for a win, 0.5 points for a draw, and 0 points for a loss in a game. It is known that the winner beat all her opponents, and her score was $\frac{1}{5}$ of the sum of the scores of all the rest of the competitors. How many competitors were in the tournament?

(A) 8 (B) 10 (C) 11 **(D$*$)** 12 (E) 14

Solution

Let n be the number of competitors. Each competitor played exactly once against any other competitor. So, they played $\frac{n(n-1)}{2}$ games. Also, there were

$$\frac{n(n-1)}{2} - (n-1) = \frac{(n-1)(n-2)}{2}$$

games in which the winner did not play. Each game is played for 1 point which means that the winner scored $n-1$ points. The rest competitors scored $\frac{(n-1)(n-2)}{2}$ points. By the conditions, we obtain the following equation:

$$\frac{(n-1)(n-2)}{2} = 5(n-1).$$

Since $n > 1$, we obtain $n = 12$.

Answer: D

7. Let x, y and z be three different positive whole numbers. Find the largest possible difference between the sum

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

and the sum

$$\frac{1}{x+1}+\frac{1}{y+2}+\frac{1}{z+3}.$$

(A) $\frac{11}{12}$ (B) $\frac{21}{20}$ (C) $\frac{13}{15}$ (D) $\frac{5}{4}$ (**E***) $\frac{13}{12}$

Solution

The difference between the sums equals

$$\frac{1}{x}-\frac{1}{x+1}+\frac{1}{y}-\frac{1}{y+2}+\frac{1}{z}-\frac{1}{z+3}$$
$$=\frac{1}{x(x+1)}+\frac{2}{y(y+2)}+\frac{3}{z(z+3)}$$

The value of the last expression will be the largest when the denominators are the smallest. Since the three smallest distinct positive integers are 1, 2 and 3, we compare the sum for all possible (x, y, z):

$$(1,2,3),\ (2,1,3),\ (1,3,2),\ (3,1,2),\ (2,3,1),\ \text{and}\ (3,2,1).$$

The largest value of the difference is $\frac{13}{12}$ for $x = 3$, $y = 2$ and $z = 1$.

Answer: E

8. The absolute value of a number a, denoted $|a|$, is the distance from the number a to 0 in the real line. For example, $|7.5| = 7.5$, $|0| = 0$ and $|-2| = 2$.

How many ordered pairs of integers (x, y) satisfy the following inequality?

$$|x| + |y| < 2026$$

(A) 2025 (B) 4051 (C) 2,049,301 (**D***) 8,205,301 (E) 12,295,801

Solution

First count all solutions where $x > 0$ and $y > 0$.

For $x = 1$ we have $y = 1, 2, 3, \ldots, 2024$ (so, 2024 pairs).

For $x = 2$, $y = 1, 2, 3, \ldots, 2023$ (so, 2023 pairs).

For $x = 3$, $y = 1, 2, 3, \ldots, 2023$ (so, 2022 pairs)

....

For $x = 2023$, $y = 1, 2$ (so, 2 pairs).

For $x = 2024$, $y = 1$ (so, 1 pair).

Therefore, the total number of solutions when $x > 0$ and $y > 0$ equals

$$1 + 2 + 3 + \cdots + 2024 = \frac{2024 \cdot 2025}{2}$$

To find the number of solutions when $x \neq 0$ and $y \neq 0$, we must multiply the last number by 4:

$$4 \cdot \frac{2024 \cdot 2025}{2} = 2 \cdot 2024 \cdot 2025$$

because we now take into account all possible signs of x and y.

Now find the number of solutions where ($x = 0$ and $y \neq 0$) or ($y = 0$ and $x \neq 0$).

For $x = 0$ we have $y = \pm 1, \pm 2, \pm 3, \ldots, \pm 2025$ (so, $2 \cdot 2025$ pairs).

For $y = 0$, $x = \pm 1, \pm 2, \pm 3, \ldots, \pm 2025$ (so, $2 \cdot 2025$ pairs).
The only solution that we have not counted is $(x, y) = (0, 0)$.

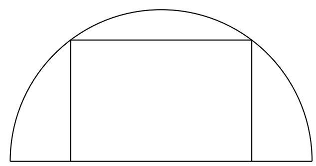
Therefore, the number of all integer solutions of the inequality equals

$$2 \cdot 2024 \cdot 2025 + 2 \cdot 2025 + 2 \cdot 2025 + 1 = 2 \cdot 2025(2024 + 1 + 1) + 1 = 2 \cdot 2025 \cdot 2026 + 1 = 8,205,301$$

Answer: $8,205,301$

Answer: D

9. In the figure the radius of the semicircle is 1.

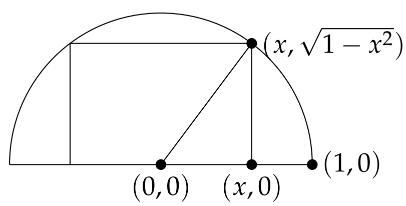


The maximum area of the rectangle is

(A) $\frac{4}{5}$ (**B***) 1 (C) $1\frac{1}{4}$ (D) $1\frac{1}{2}$ (E) 2

Solution

Let the centre of the semicircle be at $(0, 0)$ and let x be the x-coordinate of the right bottom corner of the rectangle (see the figure). Then the area of the rectangle equals $2x \cdot \sqrt{1 - x^2}$.



The value of $x \cdot \sqrt{1 - x^2}$ is maximized when its square is maximized. So consider $x^2(1 - x^2)$ instead and let us denote $x^2 = a$. Rewrite the value of $a(1 - a)$ by completing a square:

$$a(1 - a) = -a^2 + a = -a^2 + a - 0.25 + 0.25 = -(a - 0.5)^2 + 0.25.$$

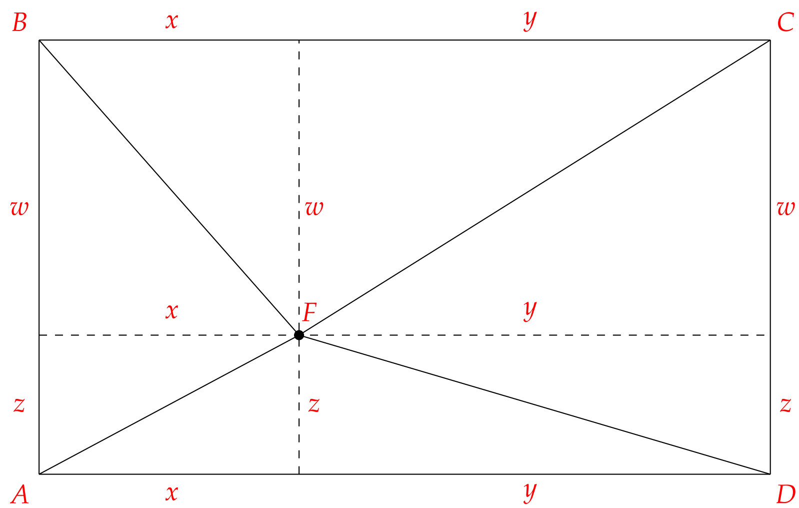
We see that the maximum value is when $a = 0.5$ which means $x = \sqrt{0.5} = \frac{1}{\sqrt{2}}$. So, the maximum area of the rectangle becomes $2\left(\frac{1}{\sqrt{2}}\right)\sqrt{1 - \frac{1}{2}} = 1$.

Answer: B

10. A fly is sitting on the ceiling of a rectangular room. The distances from the fly to the three closest corners on the floor are 3m, 4m and 5m. Find the distance from the fly to the fourth corner on the floor.

(A) 8 (B) $2\sqrt{10}$ (C) 6 (D) $5\sqrt{3}$ (**E***) $4\sqrt{2}$

Solution



The diagram shows the floor of the room where the point F is the vertical projection of the position of the fly from the ceiling to the floor. Assume that the height of the room is h and denote by x, y, z and w the distances shown in the picture. Without loss of generality, in this diagram, the distance from the fly to the bottom corners A, B and D, are 3, 4 and 5m, respectively. We need to find the distance from the fly to the bottom corner C. By the Theorem of Pythagoras,

$$\begin{cases} x^2 + z^2 = 9 - h^2 \\ x^2 + w^2 = 16 - h^2 \\ y^2 + z^2 = 25 - h^2 \end{cases}$$

From the second and third equations,

$$\begin{aligned} y^2 + w^2 &= 25 - h^2 - z^2 + 16 - h^2 - x^2 \\ &= 41 - h^2 - (h^2 + x^2 + z^2) \end{aligned}$$

By the first equation,

$$y^2 + w^2 = 41 - h^2 - 9 = 32 - h^2$$

So, the distance from the fly to the fourth corner is $\sqrt{y^2 + w^2 + h^2} = \sqrt{32} = 4\sqrt{2}$.

$\boxed{\textbf{Answer: E}}$