BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2025

Junior Preliminary Problems & Solutions

1. Suppose $d > e$, $b < e$, $c < a$, and $b > a$. The smallest of the values is:

   (A) a (B) b (**C**$*$) c (D) d (E) e

   Solution

   $$d > e > b > a > c$$

   Answer: C

2. Madame X left her entire estate to her daughter, her son, her dog, and her cat. Her daughter and son got half the estate, in a 4:3 ratio. Her dog got twice as much as her son. If the cat received \$500, then the entire estate (in dollars) was worth:

   (A) 3500 (B) 5500 (C) 6500 (**D**$*$) 7000 (E) 7500

   Solution

   Her daughter and son split half the estate into 7 parts where the son got 3 parts. The other half for the pets could be similarly split into 7 parts with the dog getting 6 parts (twice as much as her son), leaving 1 part for the cat. If \$500 was the cat's inheritance, then $14 \times 500 = 7000$ was the total.

   Answer: D

3. A hiker is walking a trail. As she has walked 1 km and 1/2 of the remaining distance, she will still need to cover 1/3 of the entire distance and another 1 km to get to the finish. What is the total distance (in km) to be covered?

   (A) 4 (**B**$*$) 9 (C) 8 (D) 11 (E) 5

   Solution

   Let S (km) be the total distance. We obtain

   $$\left(1 + \frac{S-1}{2}\right) + \left(\frac{1}{3}S + 1\right) = S$$

   $$2 + \frac{3(S-1) + 2S}{6} = S \implies 5S - 3 = 6S - 12 \implies S = 9$$

   Answer: B

4. In a Canadian history final exam, 48 students passed and 20% of those who wrote did not. How many took the history final?

   (A) 12 (B) 24 (**C**$*$) 60 (D) 120 (E) Not a whole number

Solution

If 20% did not pass, then 80% passed, or 48 students as given. $48 \div 0.8 = 60$, so a total of 60 students wrote the history final.

Answer: C

5. As the audience at the Globule Theatre waited for the production to begin, the leading lady, Lipstick Lil, was nowhere to be seen. The word was that she was in either dressing room 1, 2, or 3, so the stage manager went to look for her. Unfortunately, outside each door was a minder from the Attitude Security Company, rather disagreeable looking fellows, who refused to let anyone into the rooms they were guarding.

   "Where's Lipstick Lil?" the stage manager asked.

   "Room 1 or 3," said the minder at room 1.

   The stage manager went to room 2. "I am looking for the leading lady."

   "Room 2 or 3," said the minder.

   The minder of room 3 gave a more helpful response. "She's in room 1 or 2," he said. "But exactly two of us minders are habitual liars."

   If a habitual liar always lies, then in what room was the elusive Lipstick Lil?

   (A) room 1 (B) room 2 (**C**$*$) room 3 (D) lack of info (E) inconsistent

Solution

Lipstick Lil is in room 3. If minder 1 lies then Lil is in room 2 and if minder 2 lies Lil is in room 1. Consider the two cases for minder 3. Suppose minder 3 tells the truth. Then Lil is in room 1 or 2 and the other two lie. However, this means that Lil is in both 1 and 2, which is not possible. So minder 3 lies, implying that Lil is in room 3 and there are not two liars altogether. This is consistent with minders 1 and 2 both being truthful.

Answer: C

6. The area of a trapezoidal field is 1400 square metres. The distance between the parallel sides is 50 metres. You want to find lengths of the parallel sides, if the number of metres in each base is a multiple of 8. The number of solutions (x, y) to this problem is:

   (A) none (B) one (C) two (**D**$*$) three (E) four

Solution

Let the bases be b_1 and b_2. Then the area of the field would be

$$\frac{(b_1 + b_2) \times 50}{2} = 1400, \quad \text{or} \quad b_1 + b_2 = 56.$$

Since $56 = 7 \times 8$, $x + y = 7$ has 3 solutions: $(1,6)$, $(2,5)$, $(3,4)$ with x taking either the larger or the smaller number.

Answer: D

7. A and B together can do a job in 2 days. B and C can do it in 4 days, A and C can do it in $2\frac{2}{5}$ days. The number of days A would take to do it alone is:

(A) 1 (B) 2.8 (**C**$*$) 3 (D) 3.2 (E) 3.5

Solution

Let A alone take a days to do the job; and similarly, B, b days, C, c days. A and B together in one day can complete $1/2$ the job, resulting in the equation:

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{2}. \tag{I}$$

Similarly for B and C, and for A and C, we get

$$\frac{1}{b}+\frac{1}{c}=\frac{1}{4} \tag{II}$$

and

$$\frac{1}{a}+\frac{1}{c}=\frac{5}{12}. \tag{III}$$

Subtracting equation II from equation I, we get

$$\frac{1}{a}-\frac{1}{c}=\frac{1}{4}.$$

Now add this equation with equation III, we get

$$\frac{2}{a}=\frac{8}{12}=\frac{2}{3}, \quad \text{so} \quad a=3.$$

Answer: C

8. The number 23 initially is written on a blackboard. Each minute the current number is being erased and instead replaced by the product of its digits multiplied by 12. What number will be on the blackboard in an hour?

(A) 10 to 100 (B) 101 to 500 (C) 501 to 1000 (D) at least 1001 (**E**$*$) less than 10

Solution

At 1 minute, we have $2\times 3\times 12=72$.

At 2 minutes, we have $7\times 2\times 12=168$.

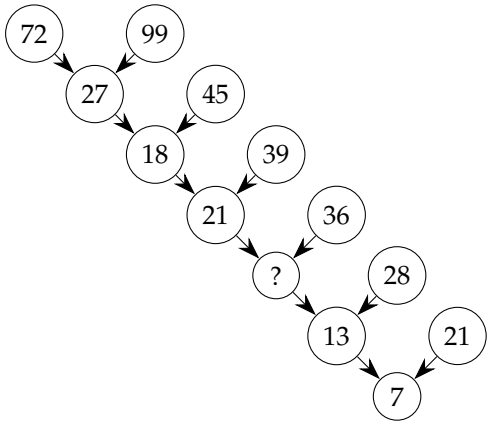
At 3 minutes, we have $1\times 6\times 8\times 12=576$.

At 4 minutes, we have $5\times 7\times 6\times 12=2520$.

Because of the 0, all the products from 5 minutes on will be 0.

Answer: E

9. Find the missing number in the following diagram to keep the pattern



(**A**$*$) 12 (B) 14 (C) 15 (D) 16 (E) 18

Solution

The diagram is obtained by taking the sum of the digits of the two numbers on the same level to get the next number, immediately on the lower level. For 21 and 36, the sum of the digits is $2+1+3+6=12$. So, the missing number is 12.

Answer: A

10. Michael and Erin measured the distance of 143 m by steps. Exactly 20 times their steps matched. Michael's step length is 65 cm. What is Erin's step length?

(**A**$*$) 55 cm (B) 50 cm (C) 52 cm (D) 45 cm (E) 44 cm

Solution

Since the distance is 14300cm and Michael's step length is 65cm, Michael made $14300/65 = 220$ steps. Let Erin's step length be h. 20 times their steps matched. The quotient $220/20 = 11$ indicates that each 11 Michael's steps equal the distance also covered by Erin by a full number of her steps. Since they matched exactly 20 times, the first time they matched is the distance $65 \times 11 = 715$cm after the start and so $\text{lcm}(65, h) = 715$. Since $715 = 5 \times 11 \times 13$, Erin's step length must be $5 \times 11 = 55$cm.

(Note, $\text{lcm}(65, 11) = 715$ but 11 is too small. Both 5 and 13 are too small, but also each of them divides 65. Further, Erin's step length cannot be $11 \times 13 = 143$ or 715 which are both unreasonably big. Also, it cannot equal $5 \times 13 = 65$, Michael's step length.)

Answer: A

11. What is the remainder when the sum of all digits of 2025! is divided by 9?

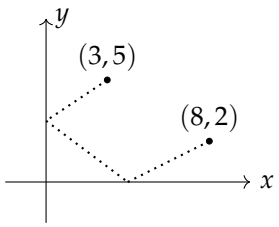
(**A**$*$) 0 (B) 1 (C) 3 (D) 4 (E) 7

Solution

Since 2025! is divisible by 9, the sum of all digits of 2025! is divisible by 9. The answer is 0.

Answer: A

12. What is the shortest path between the points $(3,5)$ and $(8,2)$ that touches each axis exactly once? An example of such a path is illustrated below.



(A) $\sqrt{90}$ (B) $\sqrt{110}$ (C) $\sqrt{130}$ (D) $\sqrt{150}$ (**E**$*$) $\sqrt{170}$

Solution

The answer doesn't change if the point $(8,2)$ is replaced with $(8,-2)$, its reflection across the x-axis. Moreover, the answer still does not change yet if $(8,-2)$ is then replaced with $(-8,-2)$, its reflection across the y-axis. After these two reflections, the shortest path is the straight line segment between the points $(3,5)$ to $(-8,-2)$, which is

$$D = \sqrt{(-8-3)^2 + (-2-5)^2} = \sqrt{170}.$$

Answer: E