BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2025

Junior Final, Part A Problems & Solutions

1. Michael wrote a test which has 20 problems. For each correctly solved problem 8 points were given. For each incorrectly solved problem 5 points were deducted. For each blank problem 0 points were given. How many problems had Michael attempted if he received 13 points?

(A) 11 (**B**$*$) 13 (C) 7 (D) 16 (E) None of the above

Solution

Since Michael scored 13 points which is not divisible by 8, he surely got incorrect answers. One can repeatedly add 5 to 13 and check whether the resulting number is divisible by 8. So,

$$13,\ 18,\ 23\ 28,\ 33,\ 38,\ 43,\ 48$$

With 48 this gives 6 correct answers, 7 incorrect answers and 7 blanks as one possibility. We check for other possibilities:

$$53,\ 58,\ 63,\ 68,\ 73,\ 78,\ 83,\ 88$$

88 is divisible by 8 but in this case the number of correct answers becomes 11 and the number of incorrect must be 15 which is too many as the sum must be 20. So Michael attempted $7 + 6 = 13$ problems.

Answer: B

2. A cargo train departed from a station at 9:00am. Later a passenger train departed from the same station at 11:00am. What distance from the station will the passenger train pass the cargo train if the speed of the cargo train is 54 km/hr and the speed of the passenger train is 72 km/hr?

(A) 156 (B) 283 (**C**$*$) 432 (D) 516 (E) 371

Solution

Let t be the time of the cargo train before the moment when the trains meet. Then the time of the passenger train is $t - 2$. Therefore,

$$54t = 72(t-2)$$

$$18t = 144 \implies t = \frac{144}{18} = 8.$$

Therefore, the cargo train will be passed by the passenger train in 8 hours since 9:00am. The distance from the station equals $S = 54 \cdot 8 = 432$ km.

Answer: C

3. The sum of 11 consecutive integers is 154. The largest of these integers is:

(A) 14 (B) 16 (C) 18 (**D**$*$) 19 (E) 20

Solution

Let n be the smallest of the integers. Then

$$n + (n+1) + (n+2) + \cdots + (n+10) = 154$$

$$11n + 55 = 154 \implies n = 9.$$

So, the largest of the integers is $9 + 10 = 19$.

Answer: D

4. "Scalene" triangles are triangles whose sides are all different lengths. Determine the number of possible scalene triangles having all sides of integral lengths, and perimeter less than 13.

(A) 1 (B) 2 (**C**$*$) 3 (D) 4 (E) 18

Solution

$\{2,3,4\}$, $\{2,4,5\}$, $\{3,4,5\}$

Answer: C

5. Nicole and Adrienne just became friends with Harmony, and they want to know when her birthday is. Harmony gives them a list of twelve possible dates:

March 1, 26
May 18, 28
June 16, 18, 20
September 1, 16, 28
November 3, 18

Harmony then tells Nicole the month and Adrienne the day of her birthday. Nicole and Adrienne then have the following conversation:

Nicole: I don't know when Harmony's birthday is, but I know that Adrienne doesn't know either.

Adrienne: At first I didn't know when Harmony's birthday is, but I know now.

Nicole: I STILL DON'T know when Harmony's birthday is.

What month is Harmony's birthday?

(A) March (B) May (C) June (**D**$*$) September (E) November

Solution

For Adrienne not to know the birthday immediately, the number could not be any of $3, 20$ or 26. Since Nicole knew this, she must have been told that the month was either May or September. Adrienne also arrives at this conclusion, and somehow determines the month. This tells Nicole that the birthday must be September 1st, September 16th, or March 28th. Since she still does not know the birthday, it must September 1st or 16th and the correct answer is (D)

Day repeated	March, September	1,1
	May, June, November	18, 18, 18
	September, May	28, 28
	June, September	16, 16
Day NOT repeated	March	26
	June	20
	November	3

Answer: D

6. Find the hundreds digit of 2025^{2025}.

(A) 1 (B) 2 (C) 3 (D) 5 (**E**$*$) 6

Solution

To find the hundreds digit, one only needs to study the power of the right most three digits 025. Since 25^n has 6 in the hundreds digit for any $n \geq 2$, the answer is 6.

$\boxed{\textbf{Answer: E}}$

7. The number 2025 can be expressed as the sum of consecutive odd numbers starting from 1:

$$2025 = 1 + 3 + 5 + \cdots + n.$$

Find n.

(A) 37 (B) 59 (C) 67 (D) 71 (**E**$*$) 89

Solution

The sum of the first n odd integers is equal to the square of n, that is

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

Then $2025 = 45^2 = 1 + 3 + 5 + \cdots + 89$, $n = 89$

<u>Alternative solution.</u> Use the formula for the sum of the first n positive integers.

$$1 + 3 + 5 + \cdots + n = 2025$$
$$2 + 4 + 6 + \cdots + (n+1) = 2025 + \frac{n+1}{2} \quad \text{(added extra } \frac{n+1}{2} \text{ ones)}$$

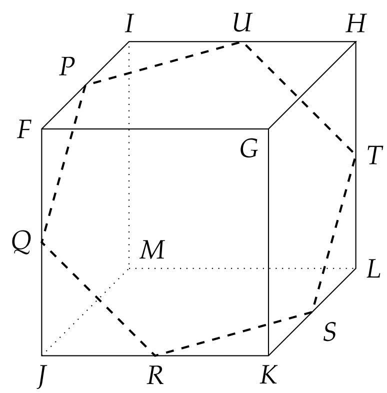
Add the equations:

$$1 + 2 + 3 + \cdots + n + (n+1) = 4050 + \frac{n+1}{2}$$
$$\frac{(n+1)(n+2)}{2} = 4050 + \frac{n+1}{2}$$
$$(n+1)^2 = 8100$$

$n + 1 = 90$ and so, $n = 89$.

$\boxed{\textbf{Answer: E}}$

8. Given a cube (with corners $FGHIMJKL$), mid-points ($PQRSTU$) of six of the twelve edges of the cube are joined to form a regular hexagon. Find the ratio of the area of the hexagon to the surface area of the cube.



(A) $\sqrt{2}:4$ (B) $\sqrt{2}:8$ (C) $\sqrt{3}:4$ (**D**∗) $\sqrt{3}:8$ (E) $\sqrt{6}:12$

Solution

Suppose the cube has centre $(0,0,0)$ and that edges are parallel with the axes. Then all of the points P, Q, R, S, T, and U lie in the plane given by $x+y+z=0$, so the hexagon lies in this plane, and the centre of the hexagon is the centre of the cube.

If the cube has side $2x$, then every side of the hexagon has length $\sqrt{2}x$. Moreover, the distance from any vertex of the hexagon to the centre is also $\sqrt{2}x$. The area of the hexagon is therefore

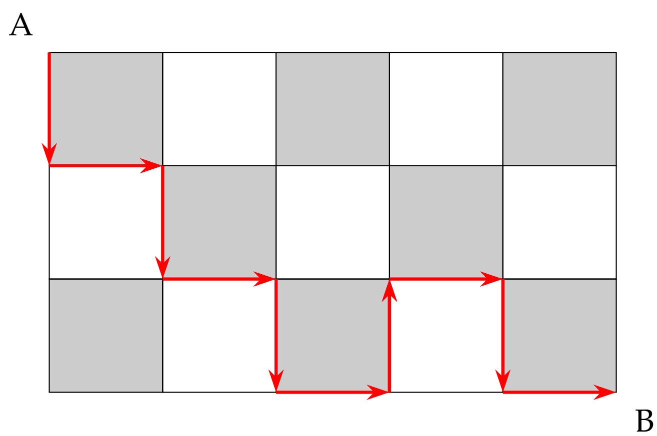
$$6 \times \tfrac{1}{2}(\sqrt{2}x)^2 \sin(\pi/3) = 6 \times \tfrac{1}{2}2x^2 \times \sqrt{3}/2 = 3\sqrt{3}x^2.$$

The area of the cube is $6(2x)^2 = 24x^2$, so the ratio is

$$3\sqrt{3}x^2 : 24x^2 = \sqrt{3} : 8$$

Answer: D

9. Fifteen tiles are arranged as shown. An ant walks along the edges of the tiles, always keeping a black tile on its left. The ant never traverses the same edge twice.

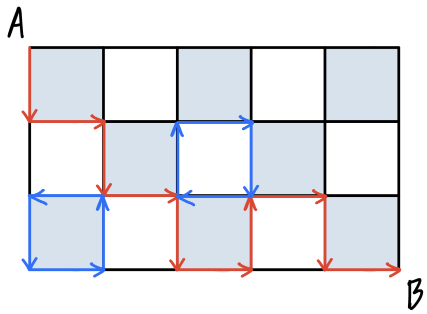


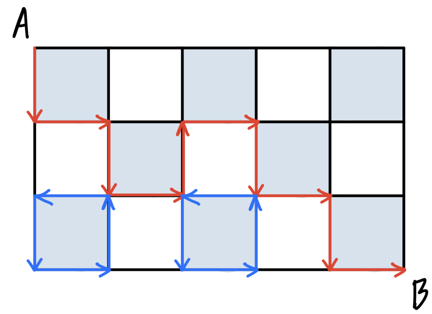
How many different routes could the ant take to get from A to B?

(**A**∗) 8 (B) 4 (C) 2 (D) 10 (E) 6

Solution

There are two distinct *paths* from A to B (shown below in red), each of which may be extended with two optional loops (shown below in blue).





To avoid double-counting, insist that blue edges are traversed with priority. Considering each blue loop as optional, there are four possible variations of each path, and hence 8 possibilities in total.

Answer: A

10. A deck of 16 cards contains 4 Jacks, 4 Queens, 4 Kings, 4 Aces. I shuffle the deck and draw 2 cards at random. I then tell you (truthfully) that I have at least one ace. What is the probability that I have two aces?

(A) $\frac{1}{5}$ (B) $\frac{1}{6}$ (**C**$*$) $\frac{1}{9}$ (D) $\frac{3}{16}$ (E) $\frac{2}{15}$

Solution

Consider the set of all ordered pairs of cards with at least one ace in each. For example, (Ace $\heartsuit$, Jack $\spadesuit$), (King $\clubsuit$, Ace $\diamondsuit$) or (Ace $\clubsuit$, Ace $\spadesuit$). Let us count all such pairs. There are $4 \times 12 \times 2 = 96$ pairs with exactly one ace and there are $4 \times 3 = 12$ of pairs with two aces. So, the total number of such pairs is $96 + 12 = 108$. Then the probability of having two aces given that there is at least one ace equals $\frac{12}{108} = \frac{1}{9}$. The answer is (C).

Answer: C