BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2025

Senior Preliminary Problems & Solutions

1. As the audience at the Globule Theatre waited for the production to begin, the leading lady, Lipstick Lil, was nowhere to be seen. The word was that she was in either dressing room 1, 2 or 3, so the stage manager went to look for her. Unfortunately, outside each door was a minder from the Attitude Security Company, rather disagreeable looking fellows, who refused to let anyone into the rooms they were guarding.

   "Where's Lipstick Lil?" the stage manager asked.

   "Room 1 or 3," said the minder at room 1.

   The stage manager went to room 2. "I am looking for the leading lady."

   "Room 2 or 3," said the minder.

   The minder of room 3 gave a more helpful response. "She's in room 1 or 2," he said. "But exactly two of us minders are habitual liars."

   If a habitual liar always lies, then in what room was the elusive Lipstick Lil?

   (A) room 1 (B) room 2 (**C**∗) room 3 (D) lack of info (E) inconsistent

   Solution

   Lipstick Lil is in room 3. If minder 1 lies then Lil is in room 2 and if minder 2 lies Lil is in room 1. Consider the two cases for minder 3. Suppose minder 3 tells the truth. Then Lil is in room 1 or 2 and the other two lie. However, this means that Lil is in both 1 and 2, which is not possible. So minder 3 lies, implying that Lil is in room 3 and there are not two liars altogether. This is consistent with minders 1 and 2 both being truthful.

   Answer: C

2. Suppose $d > e$, $b < e$, $c < a$, and $b > a$. The smallest of the values is:

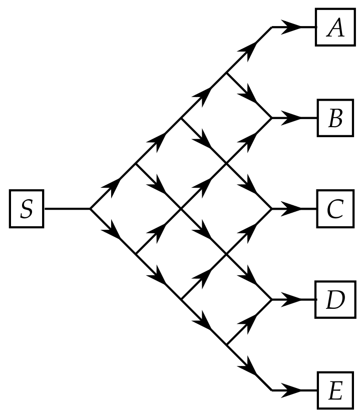
   (A) a (B) b (**C**∗) c (D) d (E) e

   Solution

   $$d > e > b > a > c$$

   Answer: C

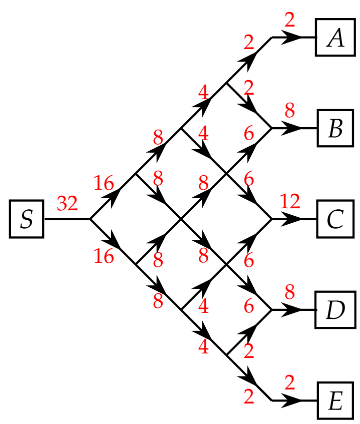
3. The map shows a system of one way trails in a park. A group of 32 hikers begins at S. Whenever two trails diverge from a point half the hikers at that point go left and half go right. All the hikers eventually reach one of the destinations A, B, C, D, or E.



The number that reach C will be

(A) 4 (B) 6 (C) 10 **(D∗)** 12 (E) 16

Solution



12 hikers will reach C.

Answer: D

4. If $z^x = y^{2x}, 2^z = 2(4^x)$ and $x + y + z = 16$, then a possible value for y is:

(A) $\frac{-3}{11}$ (B) $\frac{3}{11}$ **(C∗)** $\frac{-11}{3}$ (D) $\frac{11}{3}$ (E) -3

Solution

$$z^x = y^{2x} \implies z = y^2$$
$$2^z = 2\left(4^x\right) \implies 2^{z-1} = 4^x \implies (z-1)\ln 2 = x \ln 4 \implies (z-1)\ln 2 = 2x \ln 2 \implies z = 2x + 1$$
$$y^2 = 2x + 1 \implies x = \frac{1}{2}\left(y^2 - 1\right).$$

Then the equation $x + y + z = 16$ becomes

$$\frac{1}{2}\left(y^2 - 1\right) + y + y^2 = 16$$

or

$$1.5y^2 + y - 16.5 = 0$$

which is quadratic in y with two real roots

$$y = \frac{-1 \pm \sqrt{1 + 6 \times 16.5}}{3} = \frac{-1 \pm 10}{3} = 3, -\frac{11}{3}.$$

With the given choices, the answer is $y = -\frac{11}{3}$.

Answer: C

5. A and B together can do a job in 2 days. B and C can do it in 4 days, A and C can do it in $2\frac{2}{5}$ days. The number of days A would take to do it alone is:

(A) 1 (B) 2.8 (**C**∗) 3 (D) 3.2 (E) 3.5

Solution

Let A alone take a days to do the job; and similarly, B, b days, C, c days. A and B together in one day can complete $1/2$ the job, resulting in the equation:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2}. \qquad \text{(I)}$$

Similarly for B and C, and for A and C, we get

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{4} \qquad \text{(II)}$$

and

$$\frac{1}{a} + \frac{1}{c} = \frac{5}{12}. \qquad \text{(III)}$$

Subtracting equation II from equation I, we get

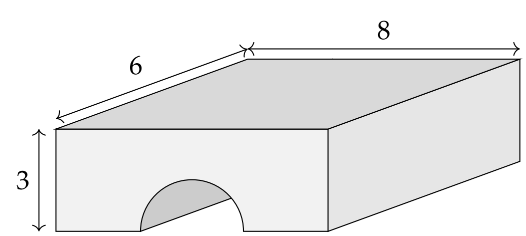
$$\frac{1}{a} - \frac{1}{c} = \frac{1}{4}.$$

Now add this equation with equation III, we get

$$\frac{2}{a} = \frac{8}{12} = \frac{2}{3}, \quad \text{so} \quad a = 3.$$

Answer: C

6. A $6 \times 8 \times 3$ rectangular prism (a box) lying flat on a table has a half-cylinder removed from the centre of the bottom (6×8). Find the diameter of the half-cylinder if its volume is 15% of the entire rectangular prism.



(**A**∗) $\frac{12}{\sqrt{5\pi}}$ (B) $\dfrac{6}{\pi\sqrt{5}}$ (C) $\dfrac{6}{\sqrt{5\pi}}$ (D) $\dfrac{24}{\sqrt{5\pi}}$ (E) $3\sqrt{\pi}$

Solution

Let r be the radius of the half-cylinder (the mousy hole). Then the volume of the hole is

$$\frac{\pi r^2}{2} \times 6 = \frac{15}{100} \times 6 \times 8 \times 3,$$

so

$$\pi r^2 = \frac{36}{5}, \quad \text{or} \quad r = \frac{6}{\sqrt{5\pi}}$$

Therefore, the diameter is $12/\sqrt{5\pi}$.

Answer: A

7. Michael and Erin measured the distance of 143 m by steps. Exactly 20 times their steps matched. Michael's step length is 65 cm. What is Erin's step length?

(**A**∗) 55 cm (B) 50 cm (C) 52 cm (D) 45 cm (E) 44 cm

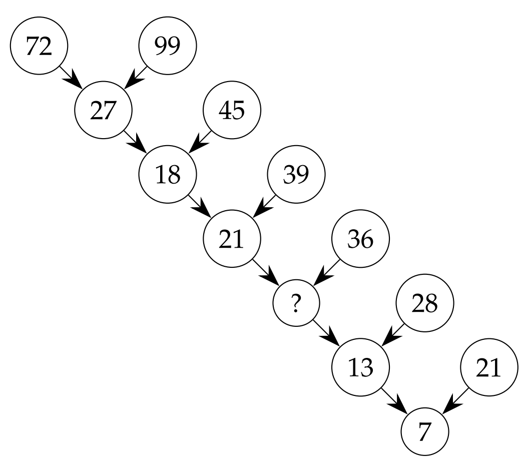
Solution

Since the distance is 14300cm and Michael's step length is 65cm, Michael made $14300/65 = 220$ steps. Let Erin's step length be h. 20 times their steps matched. The quotient $220/20 = 11$ indicates that each 11 Michael's steps equal the distance also covered by Erin by a full number of her steps. Since they matched exactly 20 times, the first time they matched is the distance $65 \times 11 = 715$cm after the start and so $\text{lcm}(65, h) = 715$. Since $715 = 5 \times 11 \times 13$, Erin's step length must be $5 \times 11 = 55$cm.

(Note, $\text{lcm}(65, 11) = 715$ but 11 is too small. Both 5 and 13 are too small, but also each of them divides 65. Further, Erin's step length cannot be $11 \times 13 = 143$ or 715 which are both unreasonably big. Also, it cannot equal $5 \times 13 = 65$, Michael's step length.)

Answer: A

8. Find the missing number in the following diagram to keep the pattern



(**A**∗) 12 (B) 14 (C) 15 (D) 16 (E) 18

Solution

The diagram is obtained by taking the sum of the digits of the two numbers on the same level to get the next number, immediately on the lower level. For 21 and 36, the sum of the digits is $2+1+3+6=12$. So, the missing number is 12.

Answer: A

9. Determine the number of trailing zeros in 2025!

Note 1) $n! = 1 \times 2 \times 3 \times \cdots \times n$

e.g. $3! = 1 \times 2 \times 3 = 6$ and $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$

Note 2) trailing zeros examples: 123<u>0000</u> has 4 trailing zeros and 10203<u>00</u> has 2 trailing zeros.

(A) 5 (B) 25 (C) 155 (**D**∗) 505 (E) 1025

Solution

Only 2×5 can provide a trailing zero. The number of the factor 2 is much higher than the number of the factor 5. So we only need to count how many factors of 5 there are.

$$\lfloor 2025/5 \rfloor + \lfloor 2025/25 \rfloor + \lfloor 2025/125 \rfloor + \lfloor 2025/625 \rfloor = 405 + 81 + 16 + 3 = 505$$

Answer: 505.

Answer: D

10. How many solutions (x, y) do there exist for the inequality $x^2 + y^2 + \frac{1}{2} \le x + y$?

(A) 0 (**B**∗) 1 (C) 2 (D) 3 (E) infinitely many

Solution

$$x^2 + y^2 + \frac{1}{2} \le x + y$$
$$x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} \le 0$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 \le 0$$

which implies $x = \frac{1}{2}$ and $y = \frac{1}{2}$. Answer: 1.

Answer: B

11. Suppose the number 123456789101112131415... is written on a board. What digit of the number is in the 2025th position?

(A) 0 (**B**∗) 1 (C) 2 (D) 5 (E) 7

Solution

The number's representation contains increasing positive integers starting from 1 with the increment of 1. To find the digit in the 2025th position, we need to find the specific integer in that location. There are 9 one-digit numbers 1 to 9, 90 two-digit numbers 10 to 99 which give the total number of digits $9 + 2 \times 90 = 189$. So, we need to use exactly $2025 - 189 = 1836$ more digits using three-digit numbers starting from 100. There will be $1836/3 = 612$ more three-digit numbers to be used from 100 to 711. So, the last digit, 1, will be in the 2025th position of the given number.

Answer: B

12. Given the line $y = \frac{3}{4}x + 6$ and a line L parallel to the given line and 4 units from it. A possible equation for L is

(A) $y = \frac{3}{4}x + 2$ (B) $y = \frac{3}{4}x$ (C) $y = \frac{3}{4}x - \frac{2}{3}$ (D) $y = \frac{3}{4}x - 1$ (**E**∗) $y = \frac{3}{4}x + 1$

Solution

The desired line L has equation $y = \frac{3}{4}x + b$ whose y-intercept, b, is to be found. The point A with the coordinates $(0, b)$, for instance, belongs to L and is 4 units away from the given line $\frac{3}{4}x - y + 6 = 0$. Writing the distance from the point A to the given line and solving for b:

$$\frac{\frac{3}{4} \cdot 0 - b + 6}{\sqrt{\frac{9}{16} + 1}} = 4$$

$$(6 - b)^2 = 16\left(\frac{9}{16} + 1\right) = 25$$

$$b = 1 \text{ or } b = 11.$$

With the provided options, $b = 1$. The line L is $y = \frac{3}{4}x + 1$.

Answer: E