BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2024
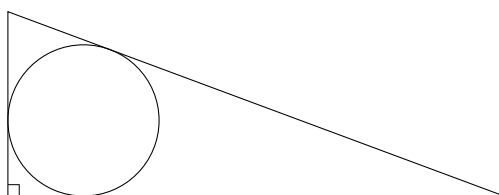
Senior Preliminary

March 2024

1. The smallest positive integer having the remainders 2, 3 and 6 when divided by 3, 5, and 11, respectively, lies between:

    (A) 41 and 50   (B) 61 and 70   (C) 71 and 80   (D) 81 and 90   (E) 91 and 100

2. The hypotenuse of a right triangle is 10 cm and the radius of the inscribed circle is 1 cm.

    The perimeter of the triangle in centimeters is:

    (A) 16   (B) 22   (C) 23   (D) 24   (E) 26

3. If $y = f(x) = \frac{x+2}{x-1}$, then it is incorrect to say:

    (A) $x = \frac{y+2}{y-1}$ (B) $f(0) = -2$ (C) $f(1) = 0$ (D) $f(-2) = 0$ (E) $f(y) = x$

4. A 25-foot ladder is placed against a wall of a building. The bottom of the ladder is 7 feet from the base of the building. If the top of the ladder slides down 4 feet, then the foot of the ladder will slide:

   (A) 9 feet   (B) 15 feet   (C) 5 feet   (D) 8 feet   (E) 4 feet

5. A rectangular box has side, front, and bottom faces with areas of $70\sqrt{3}\,\text{cm}^2$, $42\sqrt{2}\,\text{cm}^2$, and $10\sqrt{6}\,\text{cm}^2$ respectively. The volume of the box is:

    (A) $420\,\text{cm}^3$ (B) $420\sqrt{6}\,\text{cm}^3$ (C) $840\,\text{cm}^3$ (D) $420\sqrt{3}\,\text{cm}^3$ (E) $420\sqrt{2}\,\text{cm}^3$

6. All possible five-letter "words" that can be made with the letters a, b, c, d, and e are put in alphabetical order: aaaaa, aaaab, aaaac, etc. The 2024$^{\text{th}}$ "word" will be:

   (A) dabed   (B) daebd   (C) dbbae   (D) dbaed   (E) dbaee

7. Consider a $3 \times 3 \times 3$ cube composed of 27 unit cubes as an example. When an impatient painter paints the top face, paint seeps along the edges down two levels. When the same painter paints the side faces (right hand face here), paint seeps along the edges to only one level as shown in the diagrams.
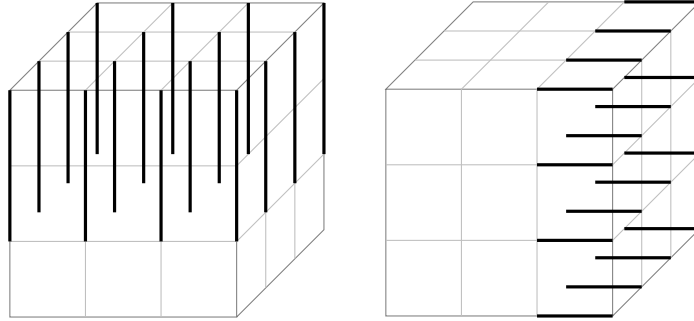
Suppose the same impatient painter paints the top face orange, the right and left faces red, and the front and back faces blue on a $5 \times 5 \times 5$ cube. (The bottom face is not accessible, so never painted.) How many unit cubes with exactly two faces painted have their edges painted by all three colours?

(A) 25   (B) 16   (C) 8   (D) 5   (E) 4

8. If $f(x) = x + 1$ and $F(x, y) = y^2 + x$ then $F(2, f(3)) =$

(A) $x^2 + 3x + 1$ (B) 19   (C) 18   (D) 8   (E) 4

9. Some time after school has ended, there are a group of students and teachers still in the gym. First, fifteen students leave, then there are two teachers per student. Then 45 teachers leave, and there are 5 students per teacher. Find the number of students in the gym at the beginning.

(A) 50   (B) 48   (C) 45   (D) 43   (E) 40

10. When riding on a train, a person counts posts that are spaced 10 meters apart alongside the tracks. For how many seconds must a person count posts in order that the number of posts counted matches exactly the numerical value of the speed of the train in km per hour?

(A) 100   (B) 60   (C) 45   (D) 36   (E) 24

11. The digits 1, 2, 3, 4, and 5 are each used once to compose a five-digit number $abcde$, such that the three digit number abc is divisible by 4, bcd is divisible by 5, and cde is divisible by 3. The digit a is:

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

12. Shown is a 21×21 square with diagonal zigzags alternating with horizontal or vertical segments beginning with lengths 1 (horizontal), 2 (vertical), 3 (horizontal), 4 (vertical), etc. Using the same pattern for a 105×105 square, find the length of the dark line that starts at the upper left hand corner and ends at the lower right hand corner.
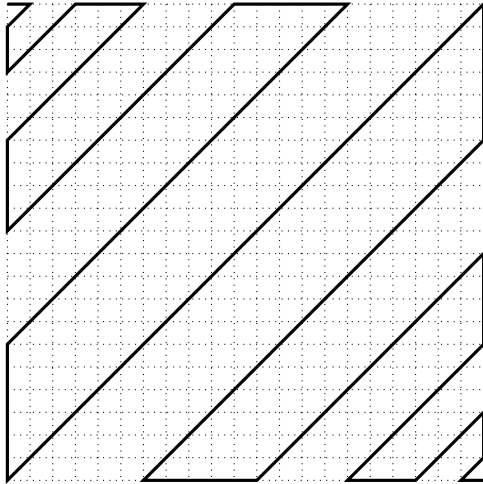
(A) $210 + 1120\sqrt{2}$
(B) $210 + 1105\sqrt{2}$
(C) $210 + 1100\sqrt{2}$
(D) $210 + 1015\sqrt{2}$
(E) $210 + 560\sqrt{2}$