# BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2024

## Senior Final, Part B

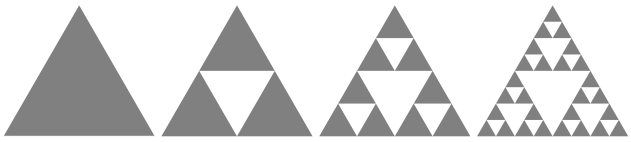
**May 3, 2024**

1. Find the smallest number such that the sum of the cubes of its digits is not divisible by the sum of its digits. Explain.

2. Determine the number of positive integral solutions of the equation $a^2 - 7a + b^2 - 7b + 2ab = 0$.

3. Let f be a real-valued function such that $f(m+n) = f(m)f(n)$. If f(4)=256 and f(k)= 0.0625, find the value of $k$.

4. The Sierpinski triangle is a self-similar fractal. It is obtained as an equilateral triangle with repeatedly removed smaller middle equilateral triangles from the remaining area. We start with a shaded equilateral triangle (stage 0), then remove the middle triangle (stage 1). For each of the three remained shaded triangles we remove the corresponding middle triangles (stage 2) and so on. The figure below shows the first three stages following stage 0.



   *a*) Find an expression (in terms of $n$) for $s$, the number of shaded triangles for stage $n$.

   *b*) Find an expression (in terms of $n$) for $u$, the number of unshaded triangles for stage $n$.

   *c*) Let $d = s - u$ be the difference between the number of shaded triangles and the number of unshaded triangles for each given stage $n$. Find and simplify $d$.

5. The Main Street Math Symposium is a club that has more than one committee. Suppose that

   - each committee consists of 4 members from the club.
   - every pair of club members serves on exactly one committee together, and
   - each pair of committees has at least one member in common.

   *a*) Show that every two committees have exactly one member in common.

   *b*) Show that each person is on at least 4 committees.

   *c*) Show that each person is on at most 4 committees.

   *d*) How many members of the club are there?