BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2024

Senior Final, Part A

May 3, 2024

1. If $f(n)$ is the minimum value of $f(x) = 2^{x^2-6x}$, then the value of $\frac{n}{f(n)}$ is:

   (A) 0 (B) 1 (C) 8 (D) -8 (E) 3×2^9

2. A trombone has been broken! Three students are being questioned by their music teacher. They each make comments about each other. Anglin says, "Ethan is lying." Ethan says, "Xavier is lying." Xavier says "both Anglin and Ethan are lying." Who is lying?:

   (A) only Anglin (B) only Ethan (C) only Xavier (D) both Anglin and Ethan

   (E) both Anglin and Xavier

3. Which of the following is a divisor of $x^{17} - 4x^{15} - x^3 + 8$?

   (A) $x + 3$ (B) $x + 2$ (C) $x - 2$ (D) $x + 1$ (E) $x - 1$

4. If $f(x) = 10^x$, then $f(x + 1) - f(x) =$

   (A) 10 (B) 90 (C) $f(x)$ (D) $f(x) - 1$ (E) $9f(x)$

5. The absolute value of x, written as $|x|$, is the distance of x from 0. For example, $|-5| = 5$, and $|3| = 3$. Determine the number of solutions of $|x||y||z| = 12$, such that x, y, and z are all integers.

   (A) 36 (B) 48 (C) 72 (D) 144 (E) 180

6. Consider equation (i): $x + y + z = 46$ and equation (ii): $p + q + r + s = 46$.
   Which of the following statements is true?

   (A) equation (i) can be solved with three consecutive integers

   (B) equation (i) can be solved with three consecutive even integers

   (C) equation (ii) can be solved with four consecutive integers

   (D) equation (ii) can be solved with four consecutive even integers

   (E) equation (ii) can be solved with four consecutive odd integers

7. All of 7 different books are distributed to four people: Maddie, Sabrina, Sacha and Adele. In how many ways can this be done if each person is to receive at least one book and no person is to receive more than two books?

(A) 15120 (B) 10080 (C) 7560 (D) 5040 (E) 2520

8. The sum of the first ten terms of an arithmetic sequence is four times the sum of the first five terms. If the arithmetic sequence is given by

$$a, a+d, a+2d, a+3d, \ldots$$

then $a : d$ is:

(A) 1:2 (B) 2:1 (C) 1:4 (D) 4:1 (E) 1:1

9. Integers m and n are chosen at random from the set $\{1, 2, \ldots, 10\}$ (with replacement, i.e. m and n can repeat). Find the probability that m and n satisfy the equation:

$$m^{n+1} + n^m = 2024$$

(A) $1/10$ (B) $1/20$ (C) $1/25$ (D) $1/50$ (E) $1/100$

10. Two right circular cones have the same height but different radii. Their bases lie in parallel planes and one of the cones is inverted so that the vertex of each cone is the center of the base of the other one. The cones intersect at circle C. If the areas of the bases are 400 and 900, then the area of C is:

(A) 100 (B) 144 (C) 288 (D) 350 (E) 650