BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2024

Junior Preliminary Problems & Solutions

1. The integers greater than 2 are arranged in columns as follows:

a	b	c	d	e
-	3	4	5	6
10	9	8	7	-
-	11	12	13	14
18	17	16	15	-
and		so		on

If one continues this pattern, the number 2024 will occur in the column:

(A) a (B) b (**C**$*$) c (D) d (E) e

Solution

The pattern starts over after every 8 counting numbers, at $8n + 3$. Since $2019 = 8 * 252 + 3$, the table will be

a	b	c	d	e
-	2019	2020	2021	2022
2026	2025	2024	2023	-

Alternative solution

Each row contains 4 numbers. There are 2024 integers from 3 to 2026 and we will have 2024/4=506 rows. The number 2024 will be in the 506th row which is even. In even numbered rows the order of the numbers is reversed. So, row 506 will be

a	b	c	d	e
2026	2025	2024	2023	-

2024 will be in column c.

Answer: C

2. The smallest positive integer having the remainders 2, 3 and 6 when divided by 3, 5, and 11, respectively, lies between:

(A) 41 and 50 (B) 61 and 70 (C) 71 and 80 (**D**$*$) 81 and 90 (E) 91 and 100

Solution

The integers between 41 and 100 that give the remainder of 6 on division by 11 are 50, 61, 72, 83 and 94.

The remainder of 50 on division by 5 is 0, not 3.

The remainder of 61 on division by 3 is 1, not 2.

The remainder of 72 on division by 3 is 0, not 2

The remainder of 83 on division by 3 is 2, and the remainder of 83 on division by 5 is 3.

So, the required number is 83.

Answer: D

3. An examination in three subjects, Algebra, Biology, and Chemistry, was taken by 41 students. The following table shows how many students failed in each subject, as well in the various combinations:

Subject	A	B	C	A,B	A,C	B,C	A,B,C
Number failed	12	5	8	2	6	3	1

(For instance, 5 students failed in Biology, among whom there were 3 who failed both Biology and Chemistry, and just 1 of these failed all three subjects.) The number of students who passed all three subjects is:

(A) 4 (B) 15 (**C**∗) 26 (D) 36 (E) 37

Solution

Inclusion/Exclusion Rule

$$\begin{aligned} N(A \cup B \cup C) &= N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap C) \\ &= 12 + 5 + 8 - 2 - 6 - 3 + 1 = 15 \end{aligned}$$

So, 15 students failed the test and $41 - 15 = 26$ passed.

Answer: C

4. A rhombus is a shape with exactly four sides that are all equal in length. Each side of a rhombus has length 10. The sum of the squares of the diagonals equals:

(A) 40 (B) 50 (C) 100 (D) 200 (**E**∗) 400

Solution

In a rhombus with side 10 let $2a$ and $2b$ denote the lengths of the diagonals.

Since $a^2 + b^2 = 100$, the sum of the squares of the diagonals is $4a^2 + 4b^2 = 400$.

Answer: E

5. Maddie walks her dog Clyde to a creek and returns home by the same route. They always walk 2 km/hr when going uphill, 6 km/hr when going downhill, and 3 km/hr when on level ground. If their total walking time is 2 hours, then the total distance they walked, in km, is:

(A) 3 (B) 4 (**C**∗) 6 (D) 7 (E) 12

Solution

Let x be the one-way ground level distance and let y be the one-way downhill distance. Then y is also the one-way uphill distance. The two-way distance becomes $2x + 2y$ and the 2 hour time spent for the two-way trip is expressed as follows

$$2 = 2\frac{x}{3} + \frac{y}{6} + \frac{y}{2}$$

or

$$2x + 2y = 6$$

The distance covered is 6 km.

Answer: C

6. An organization of 100 people wishes to set up a telephone call system. The initial contact person calls three other persons, each of whom calls three others, and so on, until all persons in the organization have been contacted. The maximum number of people who do NOT need to make a call is:

(A) 33 (B) 34 (C) 66 (**D**∗) 67 (E) 75

Solution

Step 1. The initial person contacts 3 other persons.

Step 2. Each of the 3 contacted persons in step 1 contact 3 other persons.

Step 3. Each of the 3 contacted persons in step 2 contact 3 other persons.

We obtain $1 + 3 + 9 + 27 = 40$ people after step 3. Among these 40 people, the last 27 did not make calls. 60 people are still to be contacted and so, if 20 out of those 27 make their calls, everyone will be contacted. So, $7 + 60 = 67$ don't have to make calls.

Answer: D

7. If $a^x = c^q = b$ and $c^y = a^z = d$ where $bd \neq 0$, then:

(**A**∗) $xy = qz$ (B) $\dfrac{x}{y} = \dfrac{q}{z}$ (C) $x^y = q^z$ (D) $x - y = q - z$ (E) $x + y = q + z$

Solution

Taking the xth root of b and the zth root of d in the two equations, we get

$$a = c^{\frac{q}{x}} = c^{\frac{y}{z}}$$

and so, $\frac{q}{x} = \frac{y}{z}$ which implies $xy = qz$.

Answer: A

8. In a carnival game, a player tosses a coin from a distance of about 5 feet onto a very large table tiled with 2×2-inch squares. If the coin, $\frac{3}{4}$ inches in diameter, lands entirely within a square, then the player wins; otherwise, the player loses. Assuming that the coin always lands on its face, the probability that a player wins is closest to:

(A) about $\frac{1}{10}$ (B) about $\frac{1}{5}$ (C) about $\frac{1}{4}$ (D) about $\frac{1}{3}$ (**E**∗) about $\frac{2}{5}$

Solution

We are interested in the square where the centre of the coin falls. The radius of the coin is $\frac{3}{8}$ inches, so the centre of the coin must be that distance from the edge of that square for the coin to be entirely inside. That restricts us to a square of length $2 - 2 * \frac{3}{8} = \frac{5}{4}$ which has area $\frac{25}{16}$. Since the area of the original square is 4, the probability is $\frac{25}{64}$ which is closest to $\frac{2}{5}$.

Answer: E

9. A rectangular $4 \times 3 \times 2$ block has its surface painted red, and then is cut into cubes with each edge 1 unit. The number of cubes having exactly one of its faces painted red is:

(A) 0 (**B**$*$) 4 (C) 8 (D) 12 (E) 16

Solution

The only cubes with exactly one face painted red must not touch an edge or a corner of the block. The 2x4 and 2x3 faces of the block are such that all 1x1 squares touch an edge or a corner. Each 3x4 face, however, has 2 "middle" squares. Answer: 4.

Answer: B

10. The number of integers between 100 and 1000 such that the sum of their digits is 10 is:

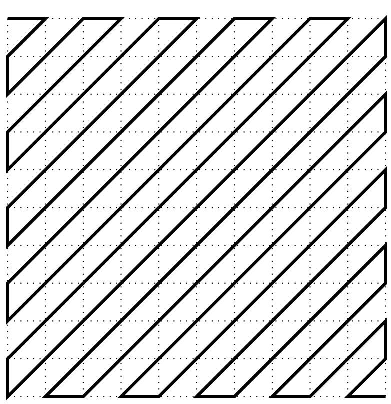
(A) 36 (**B**$*$) 54 (C) 55 (D) 62 (E) 63

Solution

If the first digit is 1, there are ten pairs for the other two digits that add up to 9. If the first digit is 2, there are nine pairs for the other two digits that add up to 8. If the first digit is 3, there are eight pairs for the other two digits that add up to 7. And so forth, noticing that if the first digit is 9, we get 901 and 910. The answer is: $10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 = 54$.

Answer: B

11. Shown is a 10×10 square. Find the length of the dark line starting at the upper left hand corner and ending at the lower right hand corner.



(A) $20 + 110\sqrt{2}$ (B) $22 + 64\sqrt{2}$ (**C**$*$) $20 + 100\sqrt{2}$ (D) $22 + 100\sqrt{2}$ (E) $20 + 90\sqrt{2}$

Solution

Half of the perimeter of the square, or 20 units, are included in the lengths. The rest of the lengths are hypotenuses of isosceles triangles with legs of lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 9, 8, 7, 6, 5, 4, 3, 2, and 1. The diagonal part is: $(1+2+3+4+5+6+7+8+9+10+9+8+7+6+5+4+3+2+1)\sqrt{2} = 100\sqrt{2}$. The answer is: $20 + 100\sqrt{2}$.

Answer: C

12. Some time after school has ended, there are a group of students and teachers still in the gym. First, fifteen students leave, then there are two teachers per student. Then 45 teachers leave, and there are 5 students per teacher. Find the number of teachers in the gym at the beginning.

(A) 40 (B) 43 (C) 45 (D) 48 (**E**$*$) 50

Solution

Let T be the number of teachers at the beginning, and S be the number of students. After 15 students leave, we have $T/2 = S - 15$. Then 45 teachers leave, so we have $5(T - 45) = S - 15 = T/2$. Solving for T, we get $T = 50$

Answer: E