# BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2024

## Junior Final, Part B

**May 3, 2024**

1. In a certain card game, one of the hands dealt contains:

   1. Exactly 13 cards.
   2. At least one card in each suit (hearts, clubs, diamonds, spades)
   3. A different number of cards in each suit.
   4. A total of five hearts and diamonds.
   5. A total of six hearts and spades.
   6. Exactly two cards in the "master" suit.

   Which of the four suits—hearts, clubs, diamonds, or spades—is the “master" suit? Explain.

2. Find the smallest number such that the sum of the cubes of its digits is not divisible by the sum of its digits. Explain.

3. Three vertices of a parallelogram in random order are $A(1,3)$, $B(4,8)$, and $C(6,2)$. Find all possible other points that can be the fourth vertex of the parallelogram.

4. The absolute value of $x$, written as $|x|$, is the distance of $x$ from 0. For example, $|-5| = 5$, and $|3| = 3$. Determine the number of solutions of $|x||y||z| = 12$ , such that $x$, $y$, and $z$ are all integers.

5. The Main Street Math Symposium is a club that has more than one committee. Suppose that

   - each committee consists of 4 members from the club.
   - every pair of club members serves on exactly one committee together, and
   - each pair of committees has at least one member in common.

   a) Show that every two committees have exactly one member in common.

   b) Show that each person is on at least 4 committees.

   c) Show that each person is on at most 4 committees.

   d) How many members of the club are there?