# BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2024

## Junior Final, Part A Problems & Solutions

1. Two logs found in a wood pile are identical in every respect. If it takes 12 seconds to cut the first log into four smaller logs, then the number of seconds required to cut the second log into five smaller logs is:

(A) 14 (B) 15 (C) 16 (D) 18 (E) 20

**Solution**

It will take 3 cuts to cut the first log into four pieces, at 4 seconds each. It will take 4 cuts to cut the second log into five pieces, for a total of 16 seconds.

$\boxed{\textbf{Answer: C}}$

2. A trombone has been broken! Three students are being questioned by their music teacher. They each make comments about each other. Anglin says, "Ethan is lying." Ethan says, "Xavier is lying." Xavier says "both Anglin and Ethan are lying." Who is lying?:

(A) only Anglin (B) only Ethan (C) only Xavier (D) both Anglin and Ethan

(E) both Anglin and Xavier

**Solution**

Suppose Anglin is telling the truth. Then Ethan must be lying, but if his statement is false, Xavier must also be telling the truth. In that case, Xavier's statement contradicts our original assumption that Anglin was telling the truth. Therefore, our assumption that Anglin is telling the truth can't be true, so Anglin must be lying. That means Ethan is telling the truth, and Xavier is also lying.

$\boxed{\textbf{Answer: E}}$

3. If all the numbers from one to one million are printed, then the number of times the digit 5 will appear is:

(A) $100,000$ (B) $200,000$ (C) $500,000$ (D) $600,000$ (E) $1,000,000$

**Solution**

By padding with leading zeros if necessary, each number may be represented as a permutation of 6 digits (e.g, represent the number 113 by 000113). Since each permutation appears exactly once, the digit in each position will be equal to 5 for exactly 10% of the permutations. Hence the digit 5 appears in each position a total of 100,000 times. Since there are 6 positions, the digit 5 appears a total of 600,000 times.

$\boxed{\textbf{Answer: D}}$

4. In Clyde's French class, each test is worth the same amount. Clyde got a 97 on his French test and raised his average from 82 to 83. In order to raise his average from 83 to 84, on the next test he must get a score of:

(A) 96 (B) 97 (C) 98 (D) 99 (E) 100

**Solution**

Let $n$ be the number of prior tests, and let $\Sigma x$ denote the sum of Clyde's scores on these prior tests. Since Clyde's original average was 82, $\Sigma x = 82n$. Since a score of 97 increased the average to 83, this implies that $\Sigma x + 97 = 83(n+1)$. Combining these equations yields $n = 14$, from which me way conclude that $\Sigma x = 82(14) = 1148$ and that there are 16 tests in total. Letting $t$ denote Clyde's score on the 16th test, in order to raise his average to 84, we must have $1148 + 97 + t = 84(16)$, which is satisfied when $t = 99$.

**Answer: D**

5. Chocolates in your favourite candy store are sold in packages of 6, 9, and 20 only. The largest number of chocolates that one cannot buy is:

(A) 29 (B) 41 (C) 43 (D) 47 (E) there is no largest number

**Solution**

Buying 44 chocolates is possible since $44 = 4 \times 6 + 1 \times 20$. Similarly, $45 = 5 \times 9$, $46 = 1 \times 6 + 2 \times 20$, $47 = 3 \times 6 + 1 \times 9 + 1 \times 20$, $48 = 8 \times 6$, $49 = 1 \times 9 + 2 \times 20$, and $50 = 5 \times 6 + 1 \times 20$.

Since the six consecutive quantities $45, \ldots, 50$ are possible, every larger quantity is possible simply by adding 6 sufficiently many times to one of the six listed.

Since 43 is not divisible by 3, it cannot be written as a sum of 6's and 9's. Similarly, $43 - 20 = 23$ cannot be written as a sum of 6's and 9's, so 43 cannot be written as the sum of a single 20 and some $6's$ and some 9's. Finally, $43 - 2 \times 20 = 3$ can clearly not be written as a sum of 6's and 9's either. Hence 43 chocolates cannot be purchased, and this is the largest number with that property.

**Answer: C**

6. The smallest possible value of the expression $\dfrac{12}{(x-1)^2+3}$ is:

(A) 4 (B) 3 (C) 2 (D) 1 (E) none of these

**Solution**

The denominator is always positive, and can may grow arbitrarily large. Hence the quotient may become arbitrarily close to 0 and has no minimum.

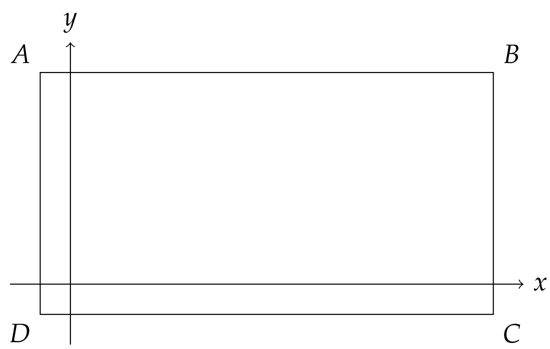
**Answer: E**

7. Suppose $ABCD$ is a rectangle with coordinates $A(-1,7)$, $B(p,7)$, $C(p,-1)$ and $D(-1,-1)$, and area 120. Then $p$ is:

(A) 14 (B) 15 (C) 16 (D) 24 (E) 25

**Solution**

The rectangle $ABCD$ has base $p+1$ and height 8, and hence area $8(p+1)$, which is equal to 120 when $p = 14$.




**Answer: A**

8. The length of each edge of a cube is increased by 50 percent. The percent increase in the surface area of the cube is:

(A) 50 (B) 125 (C) 150 (D) 300 (E) 750

**Solution**

The surface area of a cube with side length $x$ is given by $6x^2$. If the side length is increased to $x + 0.5x = 1.5x$, then the resulting surface area is $6(1.5x)^2 = 6(2.25)x^2 = 6x^2 + 1.25(6x^2)$.

**Answer: B**

9. Madeleine was born in the 19th century (between 1800 and 1899). She was $x$ years old on her birthday in the year $x^2$. On her birthday in 1875 her age in years was:

(A) 43 (B) 49 (C) 26 (D) 69 (E) not enough information

**Solution**

$43^2 = 1849$ is the only perfect square between 1800 and 1899. So the woman was 43 years old in 1849, and hence must have been 69 in 1875.

**Answer: D**

10. Two of the three altitudes of a right triangle are of lengths 4 and 5. The longest possible length of the third altitude is:

(A) $\frac{20}{3}$ (B) $\frac{20}{\sqrt{41}}$ (C) 5 (D) 6 (E) $\frac{10}{3}$

**Solution**

Suppose the area of the triangle is $A$ and that the longest possible altitude is $x$. Since the area of a triangle is $A = \frac{1}{2}bh$, where $b$ is the side length and $h$ is the altitude, the side lengths of the triangle are

$$\frac{2A}{x}, \quad \frac{2A}{5}, \quad \text{and} \quad \frac{2A}{4} = \frac{A}{2}.$$

As $x$ is the longest altitude, the side lengths above are listed in increasing order. Since the triangle is right-angled,

$$\left(\frac{2A}{x}\right)^2 + \left(\frac{2A}{5}\right)^2 = \left(\frac{A}{2}\right)^2,$$

so $\frac{4A^2}{x^2} + \frac{4A^2}{25} = \frac{A^2}{4}$, so $\frac{4A^2}{x^2} = \frac{A^2}{4} - \frac{4A^2}{25} = \frac{9A^2}{100}$, so $400A^2 = 9A^2x^2$, so $\frac{400}{9} = x^2$, so $x = \frac{20}{3}$.

**Answer: A**