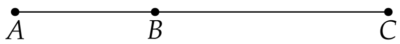
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2019

Junior Final, Part A Problems & Solutions

1. On the line segment AC shown, $AB = 12$ and $AB : BC = 3 : 5$.



The length of AC is:

(A) 16 (B) 20 (C) 24 (D) 32 (E) 60

Solution

$$\frac{12}{BC} = \frac{3}{5} \implies BC = 20 \implies AC = AB + BC = 12 + 20 = 32.$$

The correct answer is (D).

2. A statistician found the average of 43 numbers to be x. Then, by accident, she included the value x with the original numbers, and found the average of the resulting 44 numbers to be y. The ratio of y to x is:

(A) $\frac{43}{44}$ (B) $\frac{44}{43}$ (C) $\frac{45}{44}$ (D) $\frac{44}{45}$ (E) 1

Solution

Let S be the sum of the original 43 numbers. Then

$$\frac{S}{43} = x \implies S = 43x.$$

When the value x is included with the original numbers, the sum of all 44 numbers is $S + x$, so

$$\frac{S+x}{44} = y \implies S + x = 44y.$$

Subtracting these two equations gives $y = x$.

The correct answer is (E).

3. A skydiver jumps from a plane. She falls at a constant speed of 180 km/h until, half way down to the ground, she opens her parachute. After that she falls at a constant speed of 20 km/h. The total time for her descent is 5 minutes. The height of the plane when she jumped was:

(A) 3 km (B) 3.6 km (C) 4 km (D) 4.2 km (E) 10 km

Solution

Rate before $= 180\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{60\text{ min}} = 3\frac{\text{km}}{\text{min}}$

Rate after $= 20\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{3}\frac{\text{km}}{\text{min}}$

Let $T =$ time when the parachute is opened. Since the parachute was opened half the total distance down,

$$\underbrace{(\text{rate before parachute opened})}_{3} \times \underbrace{(\text{time before})}_{T} = \underbrace{(\text{rate after})}_{1/3} \times \underbrace{(\text{time after})}_{5-T}$$

$$3T = \frac{1}{3}(5 - T) \implies 9T = 5 - T \implies 10T = 5.$$

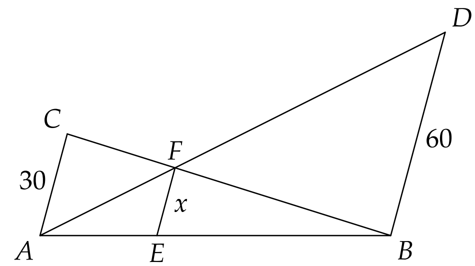
Thus $T = \frac{1}{2}$ min. Since $3T = 1.5$ km is halfway down, $2 \times 1.5 = 3$ km is the total height.

The correct answer is (A).

4. If AC, BD, and EF are parallel, then the value of x is:

(A) 10 (B) 15 (C) 20

(D) 30 (E) 45



Solution

Using similar triangles ΔEBF and ΔABC, we obtain

$$\frac{EB}{x} = \frac{AB}{30}, \qquad \text{so} \qquad EB = \frac{ABx}{30}.$$

Also using similar triangles ΔAEF and ΔABD, we obtain

$$\frac{AE}{x} = \frac{AB}{60}, \qquad \text{so} \qquad AE = \frac{ABx}{60}.$$

Hence, $AB = AE + EB = 3\frac{ABx}{60}$ and thus $x = 20$.

The correct answer is (C).

5. In a barter society among fruit growers, a lemon and 3 apples can be exchanged for 2 peaches, and an apple and 4 lemons can be exchanged for 3 peaches. The number of peaches that could be exchanged for 11 lemons is:

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

Solution

Let L, A, P be the value of a lemon, apple, or peach, respectively. We have

$$L + 3A = 2P$$

$$4L + A = 3P$$

We aim to eliminate A from these equations to obtain a relationship between L and P. Re-arranging each equation gives $3A = 2P - L$ and $A = 3P - 4L$, respectively. This implies

$$2P - L = 3A = 3(3P - 4L) = 9P - 12L \implies 7P = 11L.$$

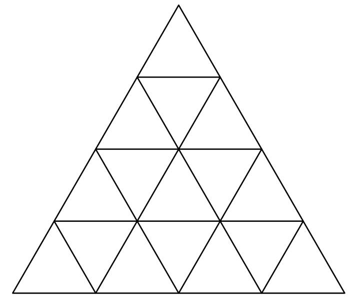
Thus 7 peaches can be exchanged for 11 lemons.

The correct answer is (A).

6. There are 27 equilateral triangles in the diagram. The largest has area 16. The sum of the areas of all 27 equilateral triangles is:

(A) 61 (B) 83 (C) 87

(D) 88 (E) 92



Solution

We have

$$16 \text{ triangles of area } 1 \text{ totals } 16 \text{ unit}^2,$$
$$7 \text{ triangles of area } 4 \text{ totals } 28 \text{ unit}^2,$$
$$3 \text{ triangles of area } 9 \text{ totals } 27 \text{ unit}^2,$$

and

$$1 \text{ triangles of area } 16 \text{ totals } 16 \text{ unit}^2$$

So,

$$27 \text{ triangles of total area} = 16 + 28 + 27 + 16 = 87 \text{ unit}^2$$

The correct answer is (C).

7. The numbers $1, 2, 3, \ldots$ are arranged in the "spiral square" pattern shown. The numbers in the row $1, 4, 15, \ldots$ are circled. The 6^{th} circled number in this sequence is:

(A) 90 (B) 92 (C) 94

(D) 96 (E) 98

$$\begin{array}{ccccccc}
10 & \rightarrow & 11 & \rightarrow & 12 & \rightarrow & 13 \\
\uparrow & & & & & & \downarrow \\
9 & & 2 & \rightarrow & 3 & & 14 \\
\uparrow & & \uparrow & & \downarrow & & \downarrow \\
8 & & \textcircled{1} & & \textcircled{4} & & \textcircled{15} \\
\uparrow & & & & \downarrow & & \downarrow \\
7 & \leftarrow & 6 & \leftarrow & 5 & & 16 \\
 & & & & & & \downarrow \\
 & \cdots & \text{etc.} & \leftarrow & 18 & \leftarrow & 17
\end{array}$$

Solution

We notice that after 2 steps, the pattern is that at each step you add what you added last time plus 8 more. That is,

$$\begin{aligned}
&1 + 3 = 4 \\
&4 + (\underbrace{3 + \mathbf{8}}_{11}) = 15 \\
&15 + (\underbrace{11 + \mathbf{8}}_{19}) = 34 \\
&34 + (\underbrace{19 + \mathbf{8}}_{27}) = 61 \\
&61 + (\underbrace{27 + \mathbf{8}}_{35}) = 96.
\end{aligned}$$

The correct answer is (D).

8. A detective questions four suspects about a crime. He takes the following statements:

Allistair: "Boris or Carmen did it."
Boris: "Allistair or Davina did it."
Carmen: "I did it."
Davina: "I didn't do it."

The 100% accurate lie-detector test indicates that three of the suspects are lying, and one of them is telling the truth. Unfortunately, the results are scrambled and it is impossible to tell which suspect is telling the truth. The crime was committed by:

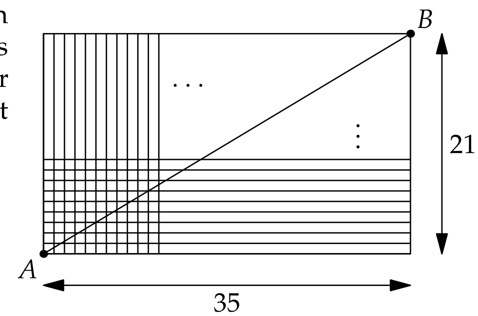
(A) Allistair (B) Boris (C) Carmen (D) Davina (E) can't be determined

Solution

Suppose Davina told the truth. In this case, Davina didn't do it and since Carmen lied, Carmen didn't do it either. So according to Boris (who lied) neither Allistair nor Davina did it. This leaves Boris as the suspect. But, this means Allistair told the truth, which is not possible. So Davina lied, which means Davina committed the crime.

The correct answer is (D).

9. A 21×35 rectangle is drawn on a grid of 1×1 squares, with vertices on intersections of the grid lines. (Not all the grid lines are shown in the diagram). A diagonal AB is drawn. The number of squares that the line segment AB will cross (the interior, not just a corner) is:



(A) 42 (B) 48 (C) 49

(D) 55 (E) 56

Solution

The slope is $\dfrac{\text{rise}}{\text{run}} = \dfrac{21}{35} = \dfrac{3}{5}$, so if A is $(0,0)$ the line passes through 7 squares and hits the point $(5,3)$.
There are 7 (3×5)-rectangles total, so it goes through $7 \times 7 = 49$ squares.
The correct answer is (C).

10. Five bags contain red and green candies as follows:

Bag A contains 2 red and 3 green candies.
Bag B contains 2 red and 4 green candies.
Bag C contains 3 red and 3 green candies.
Bag D contains 3 red and 4 green candies.
Bag E contains 5 red and 4 green candies.

You plan to take two candies at random from one of the bags. The bag that gives the highest probability that these two candies will be the same color is:

(A) Bag A (B) Bag B (C) Bag C (D) Bag D (E) Bag E

Solution

Bag A: $P(\text{Same}) = P(GG) + P(RR) = \dfrac{3}{5} \times \dfrac{2}{4} + \dfrac{2}{5} \times \dfrac{1}{4} = \dfrac{3+1}{10} = \dfrac{4}{10} = 0.4$

Bag B: $P(\text{Same}) = P(GG) + P(RR) = \dfrac{4}{6} \times \dfrac{3}{5} + \dfrac{2}{6} \times \dfrac{1}{5} = \dfrac{6}{15} + \dfrac{1}{15} = \dfrac{7}{15} = 0.46$

Bag C: $P(\text{Same}) = P(GG) + P(RR) = \dfrac{3}{6} \times \dfrac{2}{5} + \dfrac{3}{6} \times \dfrac{2}{5} = \dfrac{3}{15} + \dfrac{3}{15} = \dfrac{6}{15} = 0.4$

Bag D: $P(\text{Same}) = P(GG) + P(RR) = \dfrac{3}{7} \times \dfrac{2}{6} + \dfrac{4}{7} \times \dfrac{3}{6} = \dfrac{3}{21} + \dfrac{6}{21} = \dfrac{9}{21} = 0.43$

Bag E: $P(\text{Same}) = P(GG) + P(RR) = \dfrac{5}{9} \times \dfrac{4}{8} + \dfrac{4}{9} \times \dfrac{3}{8} = \dfrac{10}{36} + \dfrac{6}{36} = \dfrac{16}{36} = 0.44$

The correct answer is (B).