BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2014

Senior Final, Part A

Friday, May 2

1. A total of 480 students take part in an on-line game tournament. Each student participates in exactly 4 different games. Each of the games in the tournament has exactly 20 student participants and is supervised by 1 adult coach. There are 16 adult coaches and each coach supervises the same number of games. The number of games supervised by each coach is:

   (A) 16 (B) 15 (C) 12 (D) 8 (E) 6

2. An integer $n \geq 2$ is said to be **practical** if every $m = 1, 2, 3, \ldots, n-1$ can be written as a sum of some distinct divisors of n. Otherwise, n is **impractical**. For example, the divisors of 4 are 1 and 2, and $1 = 1$, $2 = 2$, and $3 = 1 + 2$, so that 4 is practical. The smallest *even* impractical number is:

   (A) 18 (B) 12 (C) 10

   (D) 6 (E) No even integer is impractical.

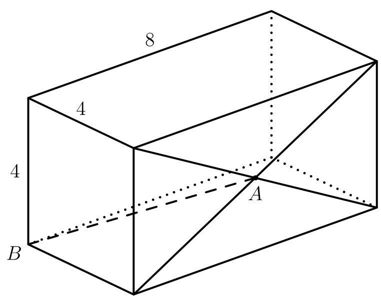
3. A regular hexagon is drawn. Then all of the non-adjacent vertices are joined to form the long and short diagonals. Let D be the set of all the line segments in the resulting figure that have two of the vertices of the original hexagon as endpoints. A pair of distinct line segments is selected at random from the set D. The probability that the two segments selected have the same length is:

   (A) $\frac{1}{35}$ (B) $\frac{1}{7}$ (C) $\frac{6}{35}$ (D) $\frac{2}{7}$ (E) $\frac{11}{35}$

4. Using only odd digits, all possible three-digit numbers are formed. The sum of all such numbers is:

   (A) 19375 (B) 62500 (C) 68750 (D) 69375 (E) 75625

5. The diagram shows the frame for a rectangular box. The ends of the frame are 4 units × 4 units squares, and the frame is 8 units long. The point A is at the point of intersection of the diagonals of one of the non-square rectangular sides. The point B is one of the corners of the box opposite the face containing point A. The length of the line segment AB is:

   (A) $4\sqrt{2}$ (B) 6 (C) $2\sqrt{6}$

   (D) 8 (E) $4\sqrt{5}$

6. Starting at the point $(0, 1)$ in the two dimensional coordinate plane draw the line to the point $(1, 0)$, and number these points 1 and 2, respectively. Starting at the point $(0, 2)$ draw the line through the points $(1, 1)$ and $(2, 0)$, and number these points 3, 4, and 5, respectively. Continuing in this way, the coordinates of point 81 are:

   (A) $(3, 9)$ (B) $(3, 10)$ (C) $(4, 9)$ (D) $(4, 10)$ (E) $(5, 9)$

7. A subterranean kingdom is completely isolated from the world on the surface, so that it is impossible for anyone who visits the kingdom to tell whether it is day or night. However, inhabitants of the kingdom have a sixth sense that allows them to tell whether it is day or night on the surface. There are two types of knights in the kingdom: day-knights and night-knights. The day-knights always tell the truth when it is day on the surface and always lie when it is night on the surface. The night-knights always tell the truth when it is night on the surface and always lie when it is day on the surface. A visitor to the kingdom encounters a knight who makes the statement "I am a day-knight and it is night on the surface." The visitor can determine:

   (A) That the speaker is a day-knight and it is day on the surface.

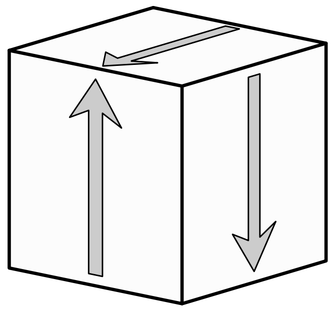
   (B) That the speaker is a day-knight and it is night on the surface.

   (C) That the speaker is a night-knight and it is day on the surface.

   (D) That the speaker is a night-knight and it is night on the surface.

   (E) Only that the speaker is a day-knight.

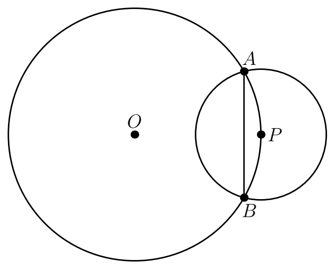
8. The cube shown has a picture of an arrow on each of its faces, each one pointing to the centre of one edge of the face. Notice that the arrows on the front and top faces meet tip-to-tip. If the three unseen arrows are oriented randomly in any of the four possible directions, the probability that no other arrows on the cube meet tip-to-tip is:



   (A) $\frac{37}{64}$ (B) $\frac{15}{32}$ (C) $\frac{27}{64}$

   (D) $\frac{7}{16}$ (E) $\frac{17}{32}$

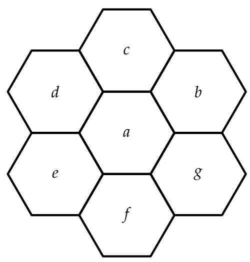
9. A circle of radius 2 has centre at point O and the chord AB has length 2. A second smaller circle has centre at point P on the first circle and goes through the points A and B. (See the diagram.) The radius of the second circle is:



   (A) $2\sqrt{2+\sqrt{3}}$ (B) $2\sqrt{2-\sqrt{3}}$ (C) 2

   (D) $2-\sqrt{3}$ (E) $2+\sqrt{3}$

10. Antonino is instructed to colour the honeycomb pattern shown, which is made up of labeled hexagonal cells. If two cells share a common side, they are to be coloured with different colours. Antonino has four colours available. Let N be the number of different ways in which he can colour the honeycomb, where two colourings are different if there is at least one cell that is a different colour in the two colourings. The value of N is:



   (A) 180 (B) 232 (C) 240

   (D) 264 (E) 288

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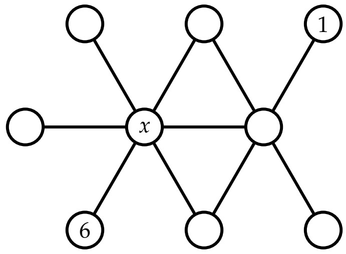
Senior Final, Part B

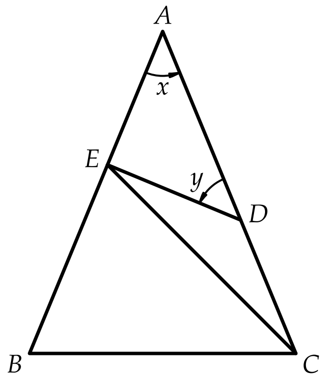
Friday, May 2

1. In the diagram, each of the integers 1 through 9 is to be placed in one circle so that the integers in every straight row of three joined circles, both horizontal and diagonal, add to 18. The 6 and 1 have been filled in as shown. Determine the value of the number represented by x in the diagram. Justify your answer.



2. A *prime number* is a positive integer with exactly two distinct factors. Two prime numbers p and q are said to be **twin primes** if $p - q = 2$. Prove that if p and q are twin primes and $p > 5$, then the integer between them must be divisible by 6.

3. Triangle ABC is isosceles with $AB = AC$ and $\angle BAC = x$. Point E is on AB with $CE = BC$, and point D is on AC with $DE = CD$. (See the diagram.) Given that $\angle ADE = y$, express y in terms of x.



4. There are ten coins each blank on one side and numbered on the other side: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. All ten coins are tossed and the sum of the numbers landing face up is calculated. What is the probability that this sum is at least 45?

5. The corners of a cube are cut off to form an equilateral triangle at each corner. The resulting solid is a truncated cube. (See the diagram.) In the truncated cube shown, the faces are regular octagons and the length of the edges of the original cube is 2 units.

   (a) Show that the side length s of the octagonal faces is given by $s = 2\left(\sqrt{2} - 1\right)$. See the diagram.

   (b) Determine the volume of the truncated cube. Note that the volume of a pyramid with base area A and height h is $V = \frac{1}{3}Ah$.