BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2022

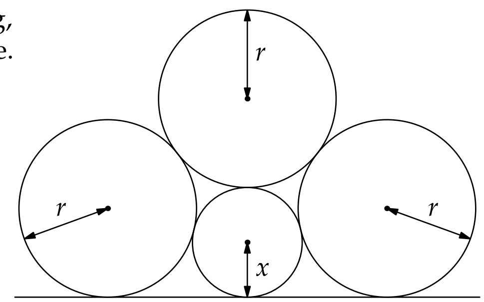
Senior Final, Part B

Friday, May 6

1. Find all the points (x, y) with $y = 3$ that form an isosceles triangle with $A(0, 0)$ and $B(5, 0)$.

2. Let n be a positive integer for which $n = a^2 + 1902$ and $n = b^2 + 2022$ for some integers a and b. Determine the highest possible value of n.

3. In the diagram shown, $r = 1$. All four circles are just touching, and each of the three bottom circles is just touching the line. Calculate x.



4. The function f defined by
$$f(x) = \frac{cx}{2x+3}, \ x \neq -\frac{3}{2},$$
where c is a real constant, satisfies $f(f(x)) = x$ for all real x except $-\frac{3}{2}$. Determine all possible values of $f(100)$.

5. Let P be a permutation of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. We define a legal operation as moving any number exactly two positions in either direction.

   For example, if $P = \{4, 2, 1, 3, 5, 6, 7, 8\}$ then we can move the number 1 two positions to the left to create $\{1, 4, 2, 3, 5, 6, 7, 8\}$, and from this new permutation we can move the number 4 two positions to the right to create $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

   We say that P is *solvable* is there exists a sequence of legal operations that takes it to the sequence $\{1, 2, 3, 4, 5, 6, 7, 8\}$. So from the above example we conclude that $\{4, 2, 1, 3, 5, 6, 7, 8\}$ is solvable.

   (a) Prove that $P = \{1, 8, 3, 2, 6, 5, 4, 7\}$ is solvable.

   (b) Prove that $P = \{1, 8, 3, 2, 6, 5, 7, 4\}$ is not solvable.

   (c) Let P be a randomly chosen permutation. Determine the probability that P is solvable.