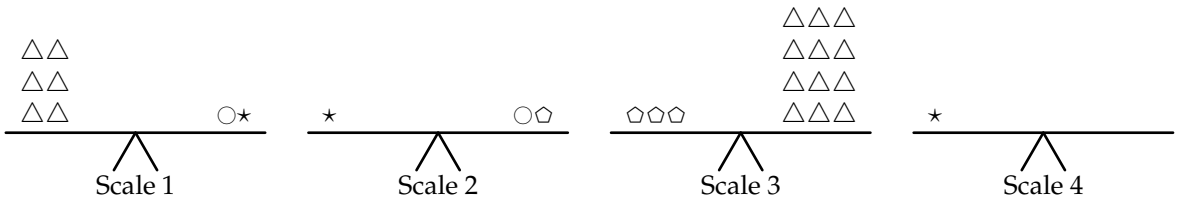
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2022

Senior Final, Part A

Friday, May 6

1. Scales 1, 2, and 3 are perfectly balanced. The number of triangles it will take to balance Scale 4, if the triangles are all placed on the right-hand side of the scale, is:



(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

2. The number of real values of x for which

$$\frac{x}{x+2} = 1 + \frac{x}{2}$$

is:

(A) 0 (B) 1 (C) 2 (D) 4 (E) infinite

3. The usual coloring pattern on an 8×8 checkerboard is changed so that 20 unit squares are now colored red, and the rest are colored white. When the board is folded in half along a line parallel to one edge of the board, exactly seven pairs of red unit squares coincide. The number of pairs of white unit squares that coincide is:

(A) 25 (B) 19 (C) 12 (D) 7 (E) 18

4. Simone plays a game starting with 5 blue marbles, 6 green marbles, and 7 red marbles. On each turn, Simone removes two marbles of different colors and adds one marble of the third color (e.g. she can remove one green and one blue and add one red.) She repeats this until only one marble is left. The final marble:

(A) must be blue (B) must be green (C) must be red

(D) can be blue or green, but not red (E) can be any color

5. Consider the sum

$$3 + 33 + 333 + \cdots + \underbrace{333333333333333}_{\text{15 digits}}.$$

The middle digit of this sum is:

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

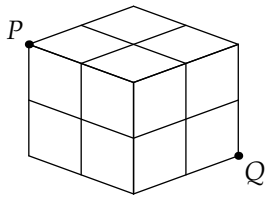
6. Eight sweaters are in a box. Each sweater has one of the numbers from 1 through 8 on it, with no duplicates. If each of 4 boys grabs a sweater out of the box, the sum of the numbers of the 4 sweaters they choose could be as low as 10 or as high as 26. The probability that the sum will be at most 13 is:

   (A) 0.05 (B) 0.1 (C) 0.15 (D) 0.2 (E) 0.25

7. Let S be the sum of all integers x such that $|4x^2 - 12x - 27|$ is a prime number. The value of S is:

   (A) -3 (B) 0 (C) 3 (D) 6 (E) 9

8. Each face of a solid cube is divided into four squares, as indicated in the diagram. Starting from vertex P, paths can be travelled to vertex Q along connected line segments. If each movement along the path must move closer to Q, the number of possible paths from P to Q is:



   (A) 36 (B) 48 (C) 54

   (D) 60 (E) 90

9. The sequence $\frac{1}{2}, \frac{5}{3}, \frac{11}{8}, \frac{27}{19}, \cdots$ is formed as follows: Each denominator is the sum of the previous numerator and denominator, and each numerator is the sum of its denominator and the denominator of the previous number. The sequence converges to the real number:

   (A) $\sqrt{3}$ (B) 2 (C) $\frac{7}{5}$ (D) $\frac{3}{2}$ (E) $\sqrt{2}$

10. Two forgetful friends have agreed to meet at a coffee shop one afternoon, but each has forgotten the agreed time of the meeting and they have no way to contact one another. Each remembers the agreed time of the meeting was between 2pm and 5pm. Each of them decides to go to the coffee shop at a random time between 2pm and 5pm, wait half an hour, then leave if the other has not arrived. The probability that they meet is:

    (A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{8}{36}$ (E) $\frac{10}{36}$