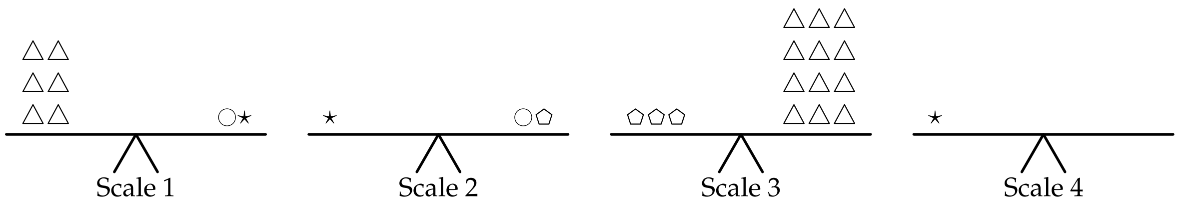
1. Scales 1, 2, and 3 are perfectly balanced.



The number of triangles it will take to balance Scale 4, if the triangles are all placed on the right-hand side of the scale, is:

(A) 4 (**B**∗) 5 (C) 6 (D) 7 (E) 8

Solution

Let T = weight of a triangle, C = weight of a circle, S = weight of a star and P = weight of a pentagon.

From scale 1: $6T = C + S$,
From scale 2: $S = C + P$,
From scale 3: $3P = 12T$ which gives $P = 4T$
so scale 2 becomes $S = C + 4T$ making scale 1: $6T = 2C + 4T$ or $2T = 2C$ or $T = C$.

Now scale 2 which was originally $S = C + P$ can be written as $S = T + 4T = 5T$.

Answer: B

2. The usual coloring pattern on an 8×8 checkerboard is changed so that 20 unit squares are now colored red, and the rest are colored white. When the board is folded in half along a line parallel to one edge of the board, exactly seven pairs of red unit squares coincide. The number of pairs of white unit squares that coincide is:

(A) 25 (**B**∗) 19 (C) 12 (D) 7 (E) 18

Solution

We know 7 pairs of red squares coincide, so 14 red squares match and 6 squares (3 pairs) are left unmatched, so they must be mixed with white squares. Out of 64 squares, 44 of them are white which are 22 pairs but 3 of the white pairs won't match because they're mixed with three pairs of red squares. The remaining white pairs that coincide are $22 - 3 = 19$ pairs.

Answer: B

3. While walking through the donkey, bird and snake houses at the zoo, Antonio has counted 35 heads and 64 feet. He knows there were half as many donkeys as birds but remembers nothing about the snakes. The number of snakes minus the number of donkeys is equal to:

(A) -3 (B) -1 (C) 0 (D) 1 (**E**∗) 3

Solution

Let D = the number of donkeys, B = the number of birds and S = the number of snakes. Counting 35 heads and 64 feet yields $D + B + S = 35$ and $4D + 2B = 64$ respectively. Donkeys being half of birds yields $D = \frac{1}{2}B$ or $B = 2D$.

$$4D + 2B = 64 \Rightarrow 4D + 4D = 64 \Rightarrow D = 8 \Rightarrow B = 16$$
$$D + B + S = 35 \Rightarrow 8 + 16 + S = 35 \Rightarrow S = 11$$

Therefore, $S - D = 11 - 8 = 3$.

Answer: E

4. A palindrome is a positive integer that reads backwards the same as it reads forwards. For example, 67276 is a palindrome.

   John thought he had added together every 2-digit positive integer, and the sum he got was a palindrome. Unfortunately, he had left one number out. The number that he left out was:

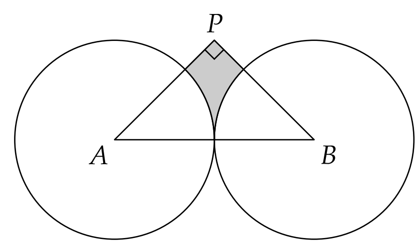
   (A) 19 (**B∗**) 21 (C) 29 (D) 31 (E) 39

Solution

The sum of all numbers from 10 to 99 is $\dfrac{90(10+99)}{2} = 4905$ which is not a palindrome. The closest palindrome to it is 4884 whose difference from 4905 is 21. So John missed the number 21.

Answer: B

5. Circles with centres A and B each have radius 2. P is equidistant from A and B, and AP is perpendicular to BP. The area of the shaded region is:

   (A) $24 - 4\pi$ (B) $8 - \pi$ (C) $6 - \pi$

   (D) $4\sqrt{2} - \pi$ (**E∗**) $4 - \pi$

Solution

$\triangle APB$ is an isosceles triangle whose hypotenuse is $AB = 2r = 4$. Using Pythagorean formula we have $AP = PB = 2\sqrt{2}$ and the area of the triangle is then $\frac{1}{2}2\sqrt{2} \cdot 2\sqrt{2} = 4$. The area of the unshaded region inside the triangle is one quarter of the area of the circle with radius 2 which is $\frac{1}{4}\pi(2)^2 = \pi$. Therefore, the area of the shaded region is $4 - \pi$.

Answer: E

6. Marbles come in 8 different colors. There are 10 marbles in a pack. The number of packs one must have to be sure of having at least 12 marbles of the same color is:

   (A) 6 (B) 7 (C) 8 (**D∗**) 9 (E) 10

Solution

Suppose that we have the maximum number of marbles possible without 12 or more of any color. For that to happen, we need groups of 11 marbles of each of the 8 colors i.e. $11 \times 8 = 88$. Now if we add one more marble then there will be at least a group of 12 marbles of the same color, so we need 89 marbles. Each pack contains 10 marbles, so 9 packs are needed for 89 marbles.

Answer: D

7. Al, Bill and Chuck were discussing their scores on a French test. Al scored 3 points below the class average, Bill 5 points above, and Chuck got 67 points. The average of the 3 boys' scores was the same as the class average. Bill's score was:

   (A) 71 (B) 72 (C) 73 (**D∗**) 74 (E) 75

Solution

Let A be the class average, then Al scored $A-3$, Bill scored $A+5$ and Chuck scored 67. Since the average of their scores is equal to the class average we must have $\dfrac{(A-3)+(A+5)+67}{3}=A$ which yields $2A+69=3A$ and $A=69$. Bill's grade is 69+5=74.

Answer: D

8. Simone plays a game starting with 3 blue marbles, 4 green marbles, and 5 red marbles. On each turn, Simone removes two marbles of different colors and adds one marble of the third color (e.g. she can remove one green and one blue and add one red.) She repeats this until only one marble is left. The final marble:

(A) must be blue (**B**∗) must be green (C) must be red

(D) can be blue or green, but not red (E) can be any color

Solution

Parity refers to whether a number is even or odd. We note that Red and Blue marbles have initially the same parity (both odd), and they will keep the same parity regardless of what move Simone makes. So to have 1 marble of one color and 0 of the others, the parity of colors must be different which implies that the color of the last marble must be green.

Answer: B

9. I have three piles of cards, with each pile containing two cards, all lying face down so you can't see the colors of the cards. One pile has two red cards, one pile has two black cards, and one pile has one red card and one black card. You don't know which pile is which. Choose a pile at random, and flip over one of the two cards. If the card you see is red, then the probability that the other card in that pile is also red is:

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{3}{5}$ (**E**∗) $\frac{2}{3}$

Solution

Label the two red cards R_1 and R_2 and the two black cards B_1 and B_2. The pairs in each pile are then R_1R_2, B_1B_2 and R_3B_3. If the card that we see is red then its accompanying card is either R_1 or R_2 or B_3, so the probability of seeing another red is $\dfrac{2}{3}$.

Answer: E

10. The greatest integer that divides $n^3(n^2-1)(n^2-4)$ for every positive integer n is:

(A) 40 (B) 60 (C) 120 (D) 180 (**E**∗) 360

Solution

The given expression is factored into $(n-2)(n-1)n^3(n+1)(n+2)$. It is the product of five consecutive integers so it's a multiple of 5, for a similar reason it is a multiple of 3 twice, a multiple of 2 and a multiple of 4, so its greatest divisor is $2\times 3^2\times 4\times 5=360$

Answer: E

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