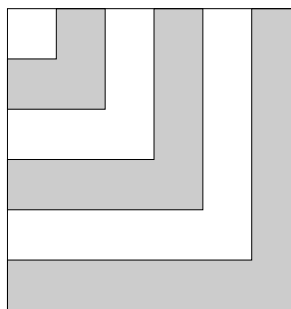


BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2023

Senior Preliminary

April 2023

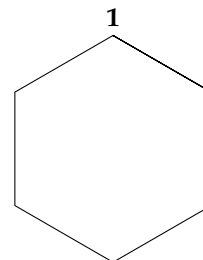
1. Two sides of a 6 cm by 6 cm square are divided into equal parts to construct the shaded and unshaded regions shown below. The ratio of shaded to unshaded area is



- (A) 3 : 1 (B) 5 : 3 (C) 7 : 5
(D) 3 : 2 (E) 6 : 5
2. Suppose $\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}} = 2023$. What is $\frac{x+y}{x-y}$?
- (A) 2023 (B) -2023 (C) 1 (D) $\frac{1}{2023}$ (E) $-\frac{1}{2023}$
3. Mary tells a secret to 6 people. Each of them tells 5 more new people. Each of them tells 4 more people. Each of them tells 3 more, each of whom tell 2 more, each of whom tells one more. At this point, how many people know the secret?
- (A) 1957 (B) 720 (C) 1237 (D) 873 (E) 1593
4. A pentagon has corners labelled A, B, C, D, E, in clockwise order. A frog starts at corner A and hops from corner to corner clockwise. On Day 1, she makes one hop, finishing the day at B. On Day 2, she makes two hops (B to C and C to D), finishing the day at D. She continues in this way, making three hops clockwise on Day 3, etc. Which corner is she at when Day 2023 ends?
- (A) A (B) B (C) C (D) D (E) E
5. You have a digital clock that shows hours and minutes, but not seconds. At one point you glance at it, and you see that the time is 1:15. Exactly 40 seconds later you glance at it again, and it says 1:16. Exactly 90 seconds after that, you glance at it again, and it says 1:17. If you glance at it again 20 seconds later, what is the probability that it will say 1:18?
- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) 1
6. Suppose $a < b < c < d < e$ are the weights of 5 pumpkins. Weighed two at a time, the weights of the pairs are 3, 5, 6, 8, 9, 11, 12, 13, 15, and 18. Then c is
- (A) 3.5 (B) 4 (C) 4.5 (D) 5 (E) 5.5

7. Determine the number of ways to position the numbers 1 through 6 on the corners of a hexagon, one number per corner, so that consecutive numbers aren't placed on adjacent corners. For example, the numbers 2 and 4 can't be on corners that connect to the corner with 3 on it. Two ways to position numbers are considered the same if one be rotated to arrive at the other. Mirror images are not considered the same.

- (A) 6 (B) 8 (C) 10
(D) 12 (E) 14



8. What is the sum of all the numbers from 1 to 100 that divisible by neither 4 nor 5?

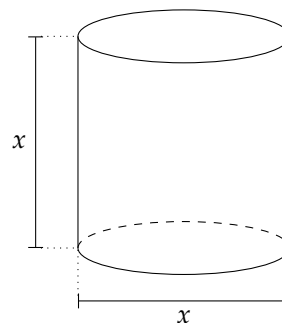
- (A) 2400 (B) 2700 (C) 3000 (D) 3300 (E) 5050

9. If $x + y = 1$ and $x^2 + y^2 = 2$, then $x^3 + y^3$ is

- (A) 1.5 (B) 2 (C) 2.5 (D) 4 (E) 8

10. Let's call a cylinder *standard* if its height equals the diameter of its (circular) base. **See diagram.** We'll say its orientation is *upright* if its top and bottom are its circular ends, and we'll say its orientation is *tipped* if it's lying on its side, so it can roll. What is the volume of the largest tipped standard cylinder that can fit inside an upright standard cylinder of volume 1? (The volume of a cylinder of radius r and height h is $\pi r^2 h$)

- (A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2\sqrt{2}}$
(D) $\frac{1}{4}$ (E) 1



11. The function $f(x)$ satisfies

$$f(x) = f(x - 1) + f(x + 1) \quad \text{for all } x.$$

If $f(1) = 4$ and $f(3) = 1$, then we can find

$$f(2) = f(1) + f(3) = 4 + 1 = 5$$

Find $f(2023)$.

- (A) -5 (B) -1 (C) 1 (D) 5 (E) 4

12. The number of triples (a, b, c) such that a, b , and c are all positive integers and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{4}$ is

- (A) 10 (B) 16 (C) 31 (D) 25 (E) 19