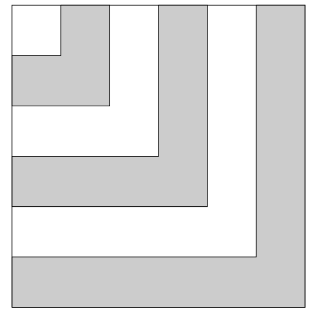
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2023

Junior Preliminary Problems & Solutions

1. Two sides of a 6 cm by 6 cm square are divided into equal parts to construct the shaded and unshaded regions shown below. The ratio of shaded to unshaded area is



(A) 3 : 1 (B) 5 : 3 (**C**∗) 7 : 5

(D) 3 : 2 (E) 6 : 5

Solution

The area of shaded region is 21 and the area of unshaded region is 15, so the ration is $\frac{21}{15} = \frac{7}{5}$.

Answer: C

2. Consider a hockey league with 7 teams. Each team plays exactly n games within the league. It is not possible for n to be:

(A) 4 (**B**∗) 7 (C) 12 (D) 14 (E) 18

Solution

If n is odd, then $7n$ (the total number of appearanances at a game) is also odd. But since each game has two teams appearing, that number should be even, so n must be even.

Answer: B

3. You have 50 coins (some quarters, the rest nickels) worth a total of \$6.10. The number of nickels is:

(A) 52 (B) 47 (C) 42 (D) 37 (**E**∗) 32

Solution

If all 50 coins were quarters, then you'd have \$12.50. That's \$6.40 too much, so we'll switch n quarters to nickels (losing 20 cents each time) \$6.40/.20 = 32 nickels.

Answer: E

4. Given the sequence $3, 12, 21, 30, 39, \ldots$, which of the following is in the sequence?

(A) 9990 (**B**∗) 9993 (C) 9994 (D) 9996 (E) 9999

Solution

The sequence is all the values that have remainder of 3 when divided by 9, and 9993 is the only choice that works.

Answer: B

5. Mary tells a secret to 6 people. Each of them tells 5 more new people. Each of them tells 4 more people. Each of them tells 3 more, each of whom tell 2 more, each of whom tells one more. At this point, how many people know the secret?

(**A**$*$) 1957 (B) 720 (C) 1237 (D) 873 (E) 1593

Solution

Build a tree diagram with Mary at the top, then 6, then $6 \times 5 = 30$, then $6 \times 5 \times 4 = 120$, then 360, then 720, then 720 again. The total: 1 + 6 + 30 + 120 + 360 + 720 + 720 = 1957.

Answer: A

6. Oliver has a basket of colored eggs. There are exactly 4 blue eggs, plus some number of red and another number of white eggs. Oliver is blindfolded, and told to take eggs out of the basket one at a time. To be certain of getting at least one white egg, he must take out 44 eggs. How many must he take out to be certain of getting at least one egg that isn't red?

(A) 5 (B) 22 (C) 39 (**D**$*$) 40 (E) 44

Solution

There must be 39 red eggs, to force the requirement that 44 are needed for a white one. Then he must have to take out 40 eggs to be certain to get something non-red.

Answer: D

7. You have a digital clock that shows hours and minutes, but not seconds. At one point you glance at it, and you see that the time is 1:15. Exactly 40 seconds later you glance at it again, and it says 1:16. Exactly 90 seconds after that, you glance at it again, and it says 1:17. If you glance at it again 20 seconds later, what is the probability that it will say 1:18?

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (**D**$*$) $\frac{2}{3}$ (E) 1

Solution

From first glance (G1), clock can be anywhere from start of 1:15 to end of 1:15. From second glance (G2), we know G1 happened between 1:15:20 and 1:16:00, and G2 happened between 1:16:00 and 1:16:40. From G3, we know G2 must have happened between 1:16:00 and 1:16:30, and G3 must have happened in the last 30 seconds of 1:17. Thus exactly 20 seconds later, there is a 2/3 chance that the clock will say 1:18.

Answer: D

8. Suppose $a < b < c < d < e$ are the weights of 5 pumpkins. Weighed two at a time, the weights of the pairs are $3, 5, 6, 8, 9, 11, 12, 13, 15$, and 18. Then $c = ?$

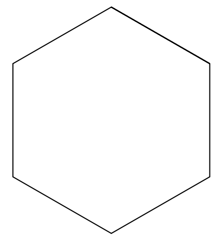
(A) 3.5 (**B**$*$) 4 (C) 4.5 (D) 5 (E) 5.5

Solution

We have $a + b = 3$. Next lightest pair must be $a + c = 5$. If we assume that a, b, c, d, e are positive integers, then we'll guess that $a = 1$, $b = 2$, $c = 4$ by now. We also have $d + e = 18$ and $e + c = 15$ which give $e = 11$ and $d = 7$.

Answer: B

9. Determine the number of ways to position the numbers 1 through 6 on the corners of a hexagon, one number per corner, so that consecutive numbers aren't placed on adjacent corners. For example, the numbers 2 and 4 can't be on corners that connect to the corner with 3 on it. Two ways to position numbers are considered the same if one be rotated to arrive at the other. Mirror images are not considered the same.



(A) 6 (B) 8 (**C**∗) 10

(D) 12 (E) 14

Solution

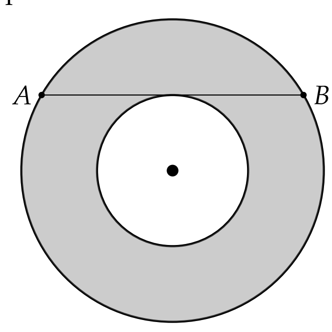
Might as well imagine hexagon with top and bottom points, with 1 at the top.

- If 2 is at the bottom, then 3 must be adjacent to 1, on either side. Then 4 is on the other side (away from 3), and can be adjacent to either 1 or 2. 5 must be between 3 and 2, and 6 must be next to 4. This gives four ways.
- If 2 is two steps away from 1 (either side) and 3 is opposite from 2, then 4 must be opposite 1 (to avoid 3, and to ensure that 5 and 6 are separated). 5 must be between 1 and 2, and 6 is between 3 and 4. This gives two more ways.
- If 2 and 3 are both two steps away from 1 (two ways to do this), then 4 must go between 1 and 2 (to avoid 3), and there are two ways to finish with 5 and 6. This gives four more ways.

  Thus there are ten ways in total.

Answer: C

10. Two circles with the same centre are shown below. Chord AB of the outer circle has length 12 and it touches the inner circle at only one point. The area of the shaded part is



(A) 6π (**B**∗) 36π (C) 72π

(D) 144π (E) 48π

Solution

Let R be the radius of the larger circle and r be the radius of the smaller circle. By a combination of symmetry and the Pythagorean Theorem, we know $r^2 + 6^2 = R^2$. The area of the shaded region is $\pi R^2 - \pi r^2 = 36\pi$.

Answer: B

11. Two ferry boats move back and forth at constant speeds across a river (taking no time to switch directions). They start at opposite sides of the river and meet 700 metres from the west side of the river. They finish their initial crossings, turn around, and meet again 400 metres from the east side of the river. The width of the river in meters is:

(A) 1000 (B) 1300 (C) 1500 (**D**∗) 1700 (E) 2100

Solution

Let $D =$ the width of the river. The first time the boats meet, they will have traveled 700 meters and $D - 700$ meters respectively. The second time they meet, they will have traveled $D + 400$ and $2D - 400$ meters respectively. Since they are traveling at constant speeds, these amounts should be proportional, i.e.: $\frac{700}{D-700} = \frac{D+400}{2D-400}$, which we can solve to find $D = 1700$ meters.

Answer: D

12. Suppose $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}} = 2023$. What is $\dfrac{x+y}{x-y}$?

(A) 2023 (**B**∗) -2023 (C) 1 (D) $\frac{1}{2023}$ (E) $-\frac{1}{2023}$

Solution

Multiply top and bottom by xy to get $\frac{y+x}{y-x} = 2023$, so answer is -2023.

Answer: B