## BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2023

## Junior Final, Part B

May 5, 2023

1. The word EUCLID can be spelled by tracing paths through the given array of letters.

Е Е D D Ε D As shown in the diagram, steps to adjacent letters  $E \longleftarrow U \longrightarrow U$ Ι Ι D horizontally, vertically, or diagonally are allowed. U L С Ι Е D Е U С L Ι D Е U U Ι T D E E Е D D D

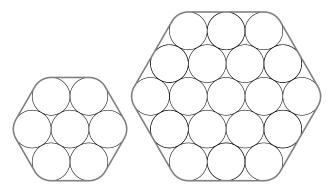
Determine the number of different paths which spell the word EUCLID.

2. How many integer solutions are there to the equation

$$\frac{P}{Q} - \frac{Q}{P} = \frac{P+Q}{PQ}$$

where  $1 \le P \le 9$  and  $1 \le Q \le 9$ .

- 3. Find the sum of all distinct four-digit numbers that contain only the digits 1, 2, 3, 4, 5 each at most once.
- 4. Suppose *n* is an integer greater than 1, that leaves the same non-zero remainder when divided into 1108, 1453, 1844, and 2258. Find *n*.
- 5. Circles of radius 1 can be arranged neatly in the shape of a hexagon, if you have an appropriate number of them. Two are shown below. The shape that is formed by an elastic stretched around the outside of one of these will be called a *circagon*, and we refer to the circagon with 6n circles touching its elastic as  $C_n$ . So  $C_1$  and  $C_2$  are the figures below.



- (a) How many circles are used to form  $C_n$ ?
- (b)What is the perimeter of  $C_n$ ? This is the length of the stretched elastic.
- (c)What is the area of  $C_n$ ? This is the area inside the stretched elastic.