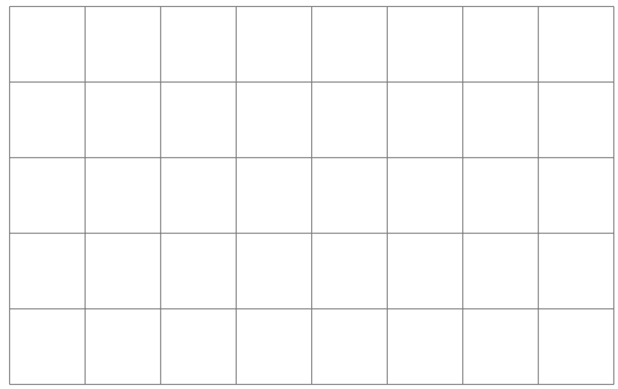
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2023

Junior Final, Part A Problems & Solutions

1. How many squares *of any size* are there in the 5 x 8 diagram below?



(A) 50

(B) 68

(C) 86

(D) 96

(**E**∗) 100

Solution

There are forty 1 x 1 squares. There are twenty eight 2 x 2 squares. There are eighteen 3 x 3 squares. There are ten 4 x 4 squares. There are four 5 x 5 squares. The total is 100 squares.

Answer: E

2. If $x^4 = 9$, then $x^{10} =$

(A) 81 (**B**∗) 243 (C) 22.5 (D) 729 (E) 486

Solution

$x^{10} = x^4 \cdot x^4 \cdot x^2 = 9 \cdot 9 \cdot 3 = 243$

Answer: B

3. You count the number of printed digits of each page in a book, and count 735 digits. (For example, there are two printed digits on page 85, and three printed digits on page 126.) How many pages are in the book?

(A) 182 (**B**∗) 281 (C) 272 (D) 245 (E) 291

Solution

Pages 1–9 give 9 digits. Pages 10–99 give 180 digits. That leaves $735 - 189 = 546$ pages with three digits. Since $546/3 = 182$, there are 182 pages with three digits, starting with 100. So the book contains 281 pages.

Answer: B

4. Three people, Pat, Nat, and Cat, know each other well. Each either always tells the truth, or always lies. They each make a statement.

- Pat: (can't be heard)

- Nat: Pat claimed to be a liar.
- Cat: Nat lied.

From the following, choose a true statement.

(A) Pat is the only one telling the truth

(B) Nat is the only one telling the truth

(**C**∗) Cat is the only one telling the truth

(D) They are all lying

(E) They are all telling the truth

Solution

Nat must be lying, since neither a liar nor a truth teller would claim to be a liar. So Cat must be telling the truth.

Answer: C

5. Oli has a rectangular sheet of plywood 135cm by 210cm. He wants to cover one side (face) with square tiles of the same size. If they need to fit exactly without overlapping or gaps, what is the *minimum* number of tiles he will need?

(A) 15   (**B**∗) 126   (C) 1134   (D) 3150   (E) 28350

Solution

Since we want the *minimum* number of same size tiles that he can use, we find the greatest common divisor of 135 and 210, which is 15. So the tiles should have side length 15, and Oli will need $14 \cdot 9 = 126$ tiles.

Answer: B

6. A five-digit number is made by randomly arranging the digits 1 to 5 with no repeats. The probability that the number is divisible by 4 is:

(**A**∗) 0.2   (B) 0.25   (C) 0.4   (D) 0.5   (E) 0.1

Solution

To determine divisibility by 4, one looks at the final two digits to see if they are divisible by 4.The number must end in 12,32,52,or 24. This accounts for $\frac{24}{120} = \frac{1}{5}$ of the possible last two digits.

Answer: A

7. The meaning of $n!$ is $n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$. Which of these is a perfect square?

(A) $\frac{23! \cdot 24!}{3}$ (B) $\frac{24! \cdot 25!}{3}$ (C) $\frac{25! \cdot 26!}{3}$ (**D**∗) $\frac{26! \cdot 27!}{3}$ (E) $\frac{27! \cdot 28!}{3}$

Solution

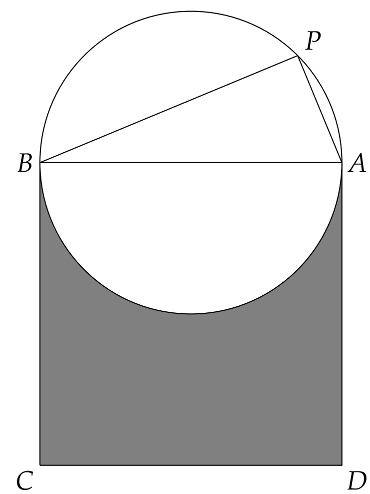
We have

$$\frac{26! \cdot 27!}{3} = 26! \cdot 26! \cdot \frac{27}{3} = (26!)^2 \cdot 3^2 = (3 \cdot 26!)^2.$$

To see that the other choices are not perfect squares, rewrite $n!(n+1)!/3$ as $n!n!(n+1)/3$.

Answer: D

8. In the diagram, AB is a diameter of the circle, and $ABCD$ is a square. If $AP = 12$ and $PB = 16$, then the area of the shaded region is:



(A) $100 - 100\pi$

(B) $400 - 100\pi$

(C) $100 - 50\pi$

(**D**∗) $400 - 50\pi$

(E) $400 - 400\pi$

Solution

Triangle APB is a right triangle, so we can use the Pythagorean theorem to find $AB = 20$. The shaded area is the area of a square of side 20 minus a semicircle of radius 10 which comes to $20^2 - \frac{\pi \cdot (10^2)}{2}$ or $400 - 50\pi$

Answer: D

9. A group of n students agreed to split the cost of a pizza that cost y dollars. If three students change their minds and decide not to, then how many more dollars will the remaining students need to contribute?

(A) $\frac{y}{n-3}$ (B) $\frac{ny}{3}$ (C) $\frac{2n+ny}{3n-n^2}$ (**D**∗) $\frac{3y}{n^2-3n}$ (E) $\frac{3y}{3n-n^2}$

Solution

Originally, each student is paying $\frac{y}{n}$ dollars, and if there are three fewer students, the rest will each need to pay $\frac{y}{n-3}$. Therefore, they will need to pay an additional $\frac{y}{n-3} - \frac{y}{n} = \frac{3y}{n^2-3n}$ dollars

Answer: D

10. Twins Lucas and Lucia are going to their cousins' house, which is 10km from their home. To save time, they share a bicycle in the following way: Lucas starts out on the bicycle, with Lucia walking behind, then he leaves the bicycle at some point and walks the rest of the way. Lucia walks until she reaches the bicycle, then rides the bicycle the rest of the way. Both can walk at a rate of 3km per hour, and bike at a rate of 10km per hour, and they reach their cousins' house at the same time. How long was the bicycle left without a rider?

(A) 1 hour

(**B**∗) 1 hour 10 minutes

(C) 1 hour 15 minutes

(D) 1 hour 20 minutes

(E) 1 hour 30 minutes

Solution

Both twins are moving for the entire time, with the same speeds walking and biking, and arrive at the same time. Therefore, the bike must have been left halfway, at the 5 km mark. Since time is distance/rate, each twin has spent 1/2 hour=30 minutes biking and 5/3 hour = 100 minutes walking, for a total of 130 minutes moving. The bike went a total of 10 km at a rate of 10 km/hr, so it was moving for 60 minutes. Therefore it was still for 130-60 = 70 minutes.

Answer: B