BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2019

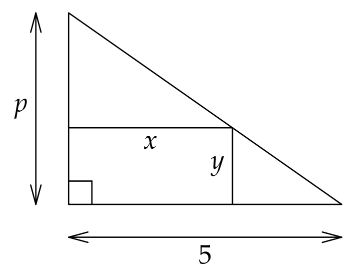
Senior Final, Part B

Friday, May 3

1. Place the numbers 1 through 10 in the empty squares in the diagram, so that the sums of the rows and the columns are as indicated. Justify why your answer is unique.

0			0	13
				17
				25
15	6	24	10	

2. A rectangle of width x and height y is inscribed in a right triangle of width 5 and height p, as shown in the diagram. Given that the rectangle has area 6, find the value of x that makes p as small as possible.



3. (a) Prove that the sum of 3 consecutive odd integers is divisible by 3.

   (b) Prove that the sum of k consecutive odd integers is divisible by k.

4. A bag contains x blue marbles and y red marbles. The numbers x and y are chosen so that if you randomly select two marbles from this bag, there is a 50% chance that the two marbles will be of the same colour.

   (a) If $y = 6$, determine all possible values of x.

   (b) Show that $x + y$ must always be a perfect square.

5. We say that a positive integer n is "special" if the first n positive integers can be partitioned into two sets, such that the sum of squares of both sets is equal. For example, $n = 12$ is special because the first 12 integers can be partitioned into sets $\{1, 2, 3, 4, 5, 7, 10, 11\}$ and $\{6, 8, 9, 12\}$, and

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 7^2 + 10^2 + 11^2 = 325 = 6^2 + 8^2 + 9^2 + 12^2.$$

   (a) Show that $n = 8$ is special.

   (b) Show that $n = 101$ is not special.

   (c) Determine whether $n = 102$ is special or not special.