BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2019

Senior Final, Part A

Friday, May 3

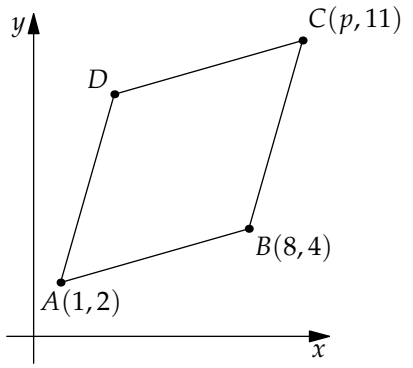
1. A statistician found the average of 43 numbers to be x. Then, by accident, she included that average x in the data set, and found the average of the 44 numbers to be y. The ratio of y to x is:

   (A) $\frac{43}{44}$ (B) $\frac{44}{43}$ (C) $\frac{45}{44}$ (D) $\frac{44}{45}$ (E) 1

2. The quadrilateral $ABCD$ shown has four sides of equal length. The value of p is:

   (A) 9 (B) 10 (C) 11

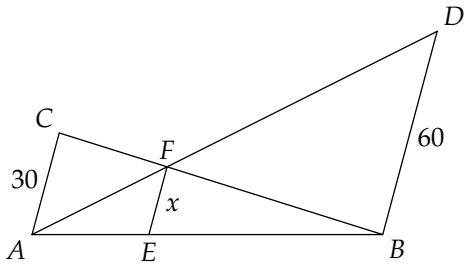
   (D) 12 (E) 15



3. If AC, BD, and EF are parallel, then the value of x is:

   (A) 10 (B) 15 (C) 20

   (D) 30 (E) 45



4. The numbers $1, 2, 3, \ldots$ are arranged in the "spiral square" pattern shown. The numbers in the row $1, 4, 15, \ldots$ are circled. The sixth circled number in this sequence is:

   (A) 90 (B) 92 (C) 94

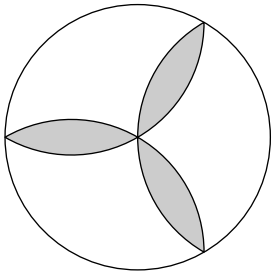
   (D) 96 (E) 98

$$\begin{array}{ccccccc} 10 & \to & 11 & \to & 12 & \to & 13 \\ \uparrow & & & & & & \downarrow \\ 9 & & 2 & \to & 3 & & 14 \\ \uparrow & & \uparrow & & \downarrow & & \downarrow \\ 8 & & \textcircled{1} & & \textcircled{4} & & \textcircled{15} \\ \uparrow & & & & \downarrow & & \downarrow \\ 7 & \leftarrow & 6 & \leftarrow & 5 & & 16 \\ & & & & & & \downarrow \\ & & & & & & 17 \end{array} \quad \cdots \text{ etc.} \leftarrow 18 \leftarrow$$

5. The equation $2x^2 + 5xy - 12y^2 = 28$ has exactly one solution in positive integers x and y. The sum of x and y is

   (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

6. The radius of the circle shown is r, and the radius of each of the arcs is r. The area of the shaded region is:



(A) $3\pi r^2$ (B) $\dfrac{3\pi r^2}{2}$ (C) $\dfrac{3\sqrt{3}r^2}{2}$

(D) $\pi r^2 - \dfrac{3\sqrt{3}r^2}{2}$ (E) $(3\sqrt{3} - \pi)r^2$

7. A detective questions four suspects about a crime. He takes the following statements:

Allistair: "Boris or Carmen did it."
Boris: "Allistair or Davina did it."
Carmen: "I did it."
Davina: "I didn't do it."

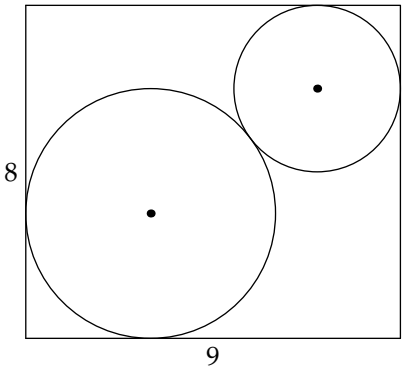
The 100% accurate lie-detector test indicates that three of the suspects are lying, and one of them is telling the truth. Unfortunately, the results are scrambled and it is impossible to tell which suspect is telling the truth. The crime was committed by:

(A) Allistair (B) Boris (C) Carmen (D) Davina (E) can't be determined

8. A box contains 10 balls numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. If three balls are randomly taken out of the box, the probability that the number on one of these balls will be the average of the other two balls is:

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{2}{15}$ (E) $\frac{1}{6}$

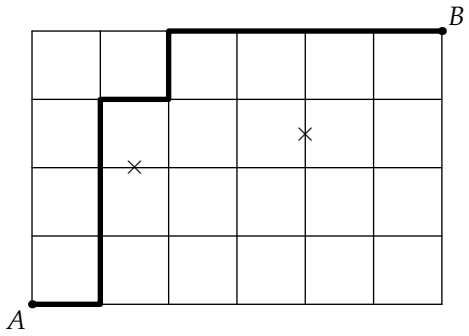
9. Two circles are inscribed in an 8×9 rectangle as shown. The sum of the radii of the circles is:



(A) $\frac{9}{2}$ (B) 5 (C) $\sqrt{26}$

(D) $\sqrt{34}$ (E) 6

10. You plan to travel from point A to point B on the grid shown, moving only up or to the right along the grid lines, and without crossing an $\times$ mark. One possible path is shown in bold. The number of possible paths is:



(A) 129 (B) 53 (C) 210

(D) 88 (E) 117