BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2019

Junior Preliminary Problems & Solutions

1. Determine the positive integer n for which $1+2+3+\cdots+n = 6+7+8$.

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Solution

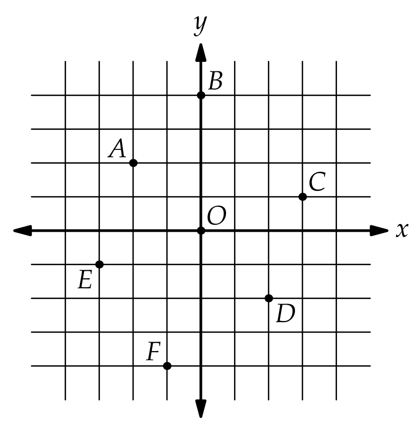
Since $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ we require

$$\begin{aligned} \frac{n(n+1)}{2} = 6+7+8 &= 21 \\ n^2+n &= 42 \\ n^2+n-42 &= 0 \\ (n-6)(n+7) &= 0 \end{aligned}$$

Therefore $n = 6$. The answer is (C).

2. Six points are in the plane, including point O at $(0,0)$ and F at $(-1,-4)$. One of the points has coordinates (x,y) where $x+y=-4$. This point is:

(A) A (B) B (C) C (D) D (E) E



Solution

$A:(-2,2)$ sum=0, $B:(0,4)$ sum=4, $C:(3,1)$ sum=4, $D:(2,-2)$ sum=0, $E:(-3,-1)$ sum=-4

The answer is (E).

3. The number $\dfrac{(2019)^2+2019(2020)}{2019+2020}$ equals:

(A) 2020 (B) 2019 (C) 4040 (D) 4038 (E) 4039

Solution

$$\frac{(2019)^2+2019(2020)}{2019+2019} = 2019\left(\frac{2019+2020}{2019+2020}\right) = 2019$$

The answer is (B).

4. Ten numbers are written on a piece of paper. Their average is 24. After two of the numbers are erased, the average of the remaining numbers is 22. The average of the two erased numbers is:

(A) 64 (B) 37 (C) 32 (D) 27 (E) 26

Solution

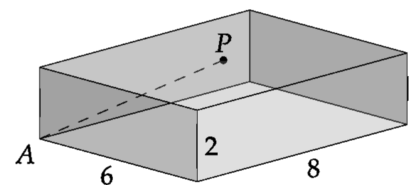
For the original 10 numbers the average is 24 so the total is $10(24) = 240$.

When we remove two, the average is 22 so the total is $8(22) = 176$.

That means that the total of the two that were erased is $240 - 176 = 64$ and their average is $64/2 = 32$.

The answer is (C).

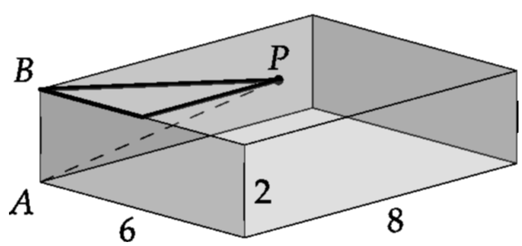
5. A rectangular room measures 2 m by 6 m by 8 m. A cord is fastened to the centre of the ceiling, P, and stretched to reach a lower corner, A. The length of the cord in metres is:



(A) $\sqrt{24}$ (B) 5 (C) $\sqrt{29}$

(D) $\sqrt{30}$ (E) none of these

Solution



Let B represent the corner directly above A. Then BP is the hypotenuse of a right triangle with sides 3 and 4 so it has length $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$. Now AP is the hypotenuse of the right triangle with sides of length 2 and 5 so it has length $\sqrt{2^2 + 5^2} = \sqrt{29}$

The answer is (C).

6. Two runners are running a 2-kilometre race on a circular 400-metre track. The ratio of their speeds is 3 : 2. The runners run in opposite directions, beginning at the same time at the start of the track. They both stop when the first person finishes. The number of times they pass each other is:

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

Solution

A 2 km race is five laps around the track. The fast runner will finish first and she will pass the slower runner every $3/5$ of the way around the track.

$$\frac{5}{3/5} = \frac{25}{3} = 8\frac{1}{3}$$

They will meet 8 times.

The answer is (A).

7. Let $n > 8$ be an integer. The remainder when $n(n+1)$ is divided by $n-2$ is:

(A) 1 (B) 2 (C) 3 (D) 6 (E) 12

Solution

We have

$$n(n+1) = n^2 + n.$$

Now use long division:

$$\begin{array}{r|ll} & n+ & 3 \\ \hline n-2 & n^2+ & n \\ & n^2- & 2n \\ \hline & & 3n \\ & & 3n- \quad 6 \\ \hline & & \quad\quad 6 \end{array}$$

The answer is (D).

Alternative solution

By the remainder theorem $2(2+1) = 6$ so the remainder is 6.

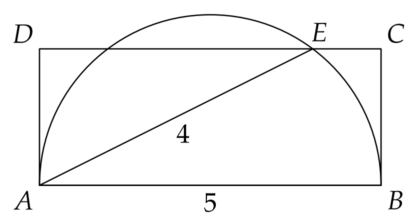
Yet another solution

In a manner similar to "completing the square" we can write

$$n^2 + n = (n-2)(n + \_\_\_) + \_\_\_$$

We can reason that the first blank must be a 3 (in order that the n term is the same on both sides), hence the second blank (the remainder) must be a 6.

8. In the figure shown, $ABCD$ is a rectangle with $AB = 5$ such that the semicircle with diameter AB cuts CD at two points, one of which is E. If $AE = 4$ then the area of $ABCD$ is:

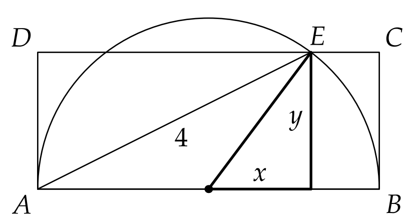


(A) 6 (B) 8 (C) 10

(D) 12 (E) 20

Solution

Drop the perpendicular (length y) from the point of intersection to the base.



Denote the distance from the point it intersects the base to the centre by x. We now have two right triangles, one with sides $x + 2.5$, y and hypotenuse 4 and the other with sides x, y and hypotenuse 2.5.

$$\begin{aligned} x^2 + y^2 &= 2.5^2 \\ (2.5 + x)^2 + y^2 &= 4^2 \\ 6.25 + 5x + x^2 + y^2 &= 16 \\ 6.25 + 5x + 2.5^2 &= 16 \\ 5x &= 3.5 \\ x &= 0.7 \\ .7^2 + y^2 &= 6.25 \\ y^2 &= 5.76 \\ y &= 2.4 \end{aligned}$$

So the area is $2.4(5) = 12$.

The answer is (D).

9. Start with a piece of paper and cut it into 8 pieces. Take any one of those 8 pieces, and cut it into 8 pieces. Take any one of *those* 8 pieces, and cut it into 8 pieces, and so on. If each time you finish cutting you count the number of pieces you have, then one possible count is:

(A) 4442 (B) 4443 (C) 4444 (D) 4445 (E) 4446

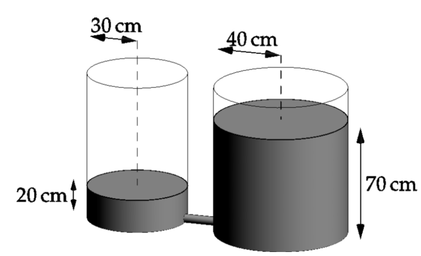
Solution

Each time you cut one piece into 8, you add 7 pieces to the total so the total number of pieces will be $7n + 1$ where n is the number of pieces you cut up.

$4442 = 634(7) + 4$ so $4446 = 634(7) + 8 = 635(7) + 1$. It is possible to get to 4446 pieces.

The answer is (E).

10. Two cylindrical tanks, one of radius 40 cm and the other of radius 30 cm, contain oil: the larger one to a depth of 70 cm, and the smaller one to a depth of 20 cm. The bottom of the tanks are at the same level and are connected by a pipe. Oil flows from one tank to the other until the depth in each tank is the same. When the oil stops flowing, the depth, in cm, of the oil in each tank will be:



(A) 52 (B) 50 (C) 48

(D) 45 (E) 43

Solution

The volume of a cylinder is $\pi r^2 h$ so the total volume of oil is

$$\pi(30^2)(20) + \pi(40^2)(70) = 18000\pi + 112000\pi = 130000\pi$$

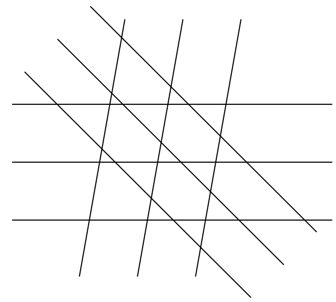
When both are filled to the same level h:

$$130000\pi = \pi(30^2)h + \pi(40^2)h = 2500\pi h$$

$$h = \frac{130000\pi}{2500\pi} = 52$$

The answer is (A).

11. Three sets of three parallel lines are shown in the figure. The number of different triangles in the figure is:



(A) 18 (B) 21 (C) 24

(D) 27 (E) 30

Solution

Every line from each group of three crosses every line from every other group of three.

Every choice of one line from each group forms a triangle so there are $3 \times 3 \times 3 = 27$ triangles, assuming no three lines intersect at the same point.

The answer is (D).

12. According to Lewis Carroll in *The Hunting of the Snark* (1876):

- All Boojums are snarks.
- Every Bandersnatch is a fruminous animal.
- Only animals which frequently breakfast at 5 o'clock tea can be snarks.
- No fruminous animals breakfast at 5 o'clock tea.

Which of the following are true?

I No Boojums are Bandersnatches
II Some snarks can be fruminous animals
III No Bandersnatches breakfast at 5 o'clock tea

(A) only I (B) only II (C) only III

(D) both I and II (E) both I and III

Solution

Every Bandersnatch is a fruminous animal.

Since no fruminous animals breakfast at 5 o'clock tea and only animals which breakfast at 5 o'clock tea can be snarks, no Bandersnatch can be a snark.

Since all Boojums are snarks, no Boojums are Bandersnatches. Therefore I is true.

Since only animals which frequently breakfast at 5 o'clock tea can be snarks and no fruminous animals breakfast at 5 o'clock tea, no fruminous animal is a snark. Therefore II is false.

Since every Bandersnatch is a fruminous animal and no fruminous animals breakfast at 5 o'clock tea III is true.

The answer is (E).