BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2018

Senior Preliminary

1. Jar A contains flour and sugar in the ratio $5:1$. Jar B, which is three times larger than Jar A, contains flour and sugar in the ratio $8:1$. When the contents of these jars are combined, the resulting mixture contains flour and sugar in the ratio $x:1$. The value of x is:

   (A) 6 (B) $\frac{13}{2}$ (C) $\frac{47}{7}$ (D) 7 (E) $\frac{29}{4}$

2. Consider the statement: "In a room with exactly N people, you are guaranteed to be able to find at least three different people who are born in the same month." The smallest possible value of N that makes this statement true is:

   (A) 24 (B) 25 (C) 35 (D) 36 (E) 37

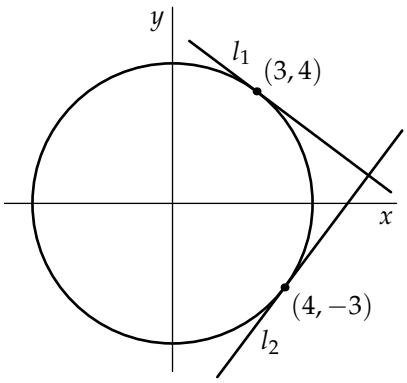
3. We say a positive integer is "happy" if it is less than 100 and is divisible by either 3 or 7, or both. For example, $3, 70$ and 84 are all happy. The number of happy numbers is:

   (A) 39 (B) 40 (C) 43 (D) 45 (E) 47

4. In the diagram shown, l_1 and l_2 are lines that are tangent to the circle $x^2 + y^2 = 25$ at the points $(3, 4)$ and $(4, -3)$. Let (p, q) be the coordinates of the point of intersection of l_1 and l_2. The sum $p + q$ is:

   (A) $\frac{15}{2}$ (B) $\frac{17}{2}$ (C) 7

   (D) 8 (E) 9



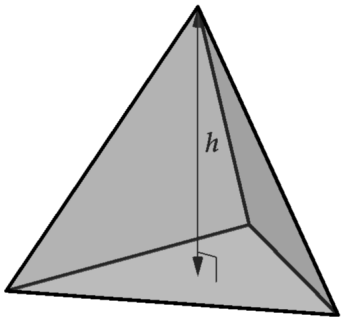
5. Joshua has chosen a 3-digit number. You are permitted to ask questions to which the answer will be "yes" or "no." The smallest number of questions you must ask Joshua in order to determine his number with certainty is:

   (A) 8 or fewer (B) 9 to 11 (C) 12 to 19 (D) 20 to 100 (E) more than 100

6. A straight line passes through three points with coordinates $(0, 12)$, $(x, 93)$ and $(100, 120)$. The value of x is:

   (A) 69 (B) 70 (C) 72 (D) 75 (E) 77

7. All four faces of the tetrahedron shown are equilateral triangles of side length 2 units. The height h of this tetrahedron is:



(A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) $\sqrt{\frac{8}{3}}$
(D) $\sqrt{8}$ (E) 3

8. Xavier, Ximena, Xander, Yolanda, Yuri, and Yosef go to math class and sit in a row of 6 seats. The probability that at least one pair whose names start with the same letter sit next to each other is:

(A) 20% (B) 50% (C) 80% (D) 85% (E) 90%

9. Suppose x and y are positive integers satisfying $x^2 = y^2 + n$. A value of n that is **not** possible is:

(A) 149 (B) 150 (C) 151 (D) 152 (E) 153

10. The remainder when $1^{2018} + 3^{2018} + 5^{2018} + 7^{2018} + 9^{2018}$ is divided by 20 is:

(A) 5 (B) 7 (C) 13 (D) 15 (E) 17

11. Each of the letters $A, B, C, D, E, F, G, H, I$, and J is assigned to exactly one digit from 0 through 9, with no two letters being assigned to the same digit. The digits have the property that $ABABFJAI \div ABC = DEFGD$, with no remainder. To the right is the completed long division. The digit assigned to H is:

$$\begin{array}{r|l} & DEFGD \\ \hline ABC & ABABFJAI \\ & AAAI \\ \hline & DIF \\ & FIF \\ \hline & AGGJ \\ & DDB \\ \hline & AAA \\ & GGG \\ \hline & AAAI \\ & AAAI \end{array}$$

(A) 1 (B) 3 (C) 5
(D) 7 (E) 9

12. A girl walks at 4 km/hr, a boy walks at 3 km/hr, and a dog runs at 6 km/hr. The girl and the boy are 2 km apart on a straight road, and the dog is midway between them. The girl follows after the boy who walks away from her, and the dog runs back and forth between the two of them. If the dog starts by running after the boy, then the number of km between the girl and the dog after one hour is:

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) $1\frac{1}{3}$