BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2017
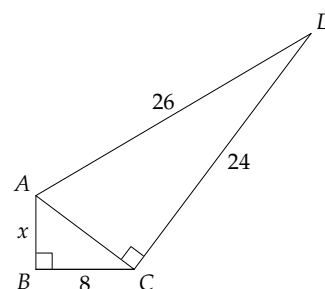
Senior Preliminary

Wednesday, April 5

1. If $3^x + 3^x + 3^x = 243$ and $2^y + 2^y = 16$, what is the value of $x + y$?

   (A) 6   (B) 7   (C) 8   (D) 9   (E) 10

2. In the figure shown, $ABCD$ is a quadrilateral with $\angle ABC = 90°$ and $\angle ACD = 90°$. If $BC = 8$, $CD = 24$, $AD = 26$ and $AB = x$, determine x.

   (A) 6   (B) 8   (C) 10

   (D) 12   (E) 15

3. A rectangular box has six faces, whose areas are 45, 45, 90, 90, 200 and 200 cm^2. Determine the volume of this box.

   (A) 450 cm^3 (B) 600 cm^3 (C) 810 cm^3 (D) 900 cm^3 (E) 1800 cm^3

4. The average age of 120 people is 35. The average age of the men is 32, while the average age of the women is 37. How many women are there?

   (A) 60   (B) 66   (C) 72   (D) 75   (E) 80

5. Four students each roll a 6-sided die (assume the numbers 1 through 6 are equally likely to occur). What is the probability that they all roll different numbers?

   (A) less than 0.15   (B) between 0.15 and 0.3   (C) between 0.3 and 0.45

   (D) between 0.45 and 0.6   (E) greater than 0.6

6. The base of a certain pyramid is a regular hexagon with side length 2 cm. Each of the sloped edges has length 4 cm as shown in the diagram. Determine the angle AVD.

   (A) 30°   (B) 45°   (C) 60°
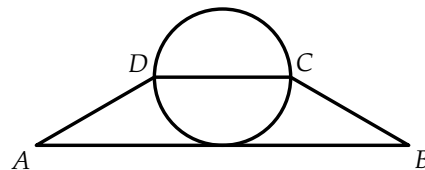
   (D) 75°   (E) 90°
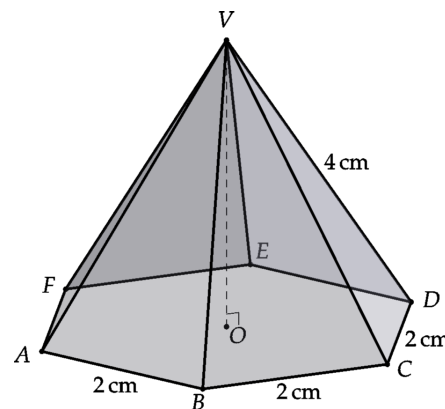
7. Recall that $n! = n \cdot (n-1)(n-2) \cdots 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Which of the following numbers is a perfect square?

   (A) $\frac{45!46!}{2}$ (B) $\frac{46!47!}{2}$ (C) $\frac{47!48!}{2}$ (D) $\frac{48!49!}{2}$ (E) $\frac{49!50!}{2}$

8. Consider a trapezoid $ABCD$ with AB parallel to CD. A circle has a diameter CD, passes through the midpoints of the diagonals AC and BD, and touches AB at one point. The angle DAB is:

   (A) $15°$ (B) $22.5°$ (C) $30°$

   (D) $36°$ (E) $45°$

9. In the xy-plane, consider the sixteen points (x, y) with x and y both integers such that $1 \le x \le 4$ and $1 \le y \le 4$ (as shown in the diagram). Determine the number of triangles with positive area whose three vertices are chosen from these sixteen points.

   (A) 496   (B) 516   (C) 520

   (D) 528   (E) 560
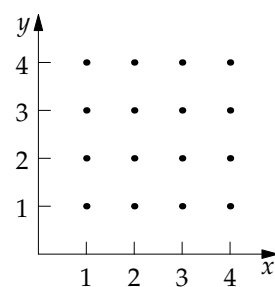
10. What is the number of pairs of positive integers (p, q) such that the equation $x^2 - px - q = 0$ has a root x with $0 < x < 8$?

    (A) 0   (B) 64   (C) 162   (D) 210   (E) 217

11. A particle moves through the the xy-plane in the following spiral formation: it starts at (0,0), then it moves 1 unit parallel to an axis in each second. The first several moves are (0,1), (1,1), (1,0), (1,-1), (0,-1), (-1,-1), (-1,0), (-1,1), (-1,2), (0,2), (1,2), (2,2), (2,1), and so on. At which point will the particle be after exactly 2017 seconds ?

    (A) $(-22, -22)$ (B) $(-22, -15)$ (C) $(-22, -5)$ (D) $(-22, 5)$ (E) $(-22, 15)$

12. If $a^2 + b^2 = c^2 + d^2 = 1$ and $ac + bd = 0$ then the maximum possible value of $ad - bc$ is:

    (A) 0   (B) $\frac{1}{2}$ (C) 1   (D) $\sqrt{2}$ (E) 2