BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2016

Senior Preliminary

Wednesday, April 6

1. Three hedgehogs (Ronny, Steph and Pat) are having a race against two tortoises (Ellery and Olly). At some point in the race Steph (S) is 10 m behind Olly (O), and Olly is 25 m ahead of Ronny (R). Ronny is 5 m behind Ellery (E), and Ellery is 25 m behind Pat (P). The order, from first to last, of the five racers at this point in the race spells the word:

   (A) PORES (B) POSER (C) PROSE (D) ROPES (E) SPORE

2. If it takes 864 digits to number the pages in a book, how many numbered pages are in the book?

   (A) 286 (B) 288 (C) 324 (D) 325 (E) 864

3. A two-digit integer is m times the sum of its digits. When the digits are reversed, the new number is n times the sum of its digits. What is the value of $m + n$?

   (A) 5 (B) 8 (C) 9 (D) 10 (E) 11

4. A wire is cut into two pieces of equal length. One is bent to form an equilateral triangle with area 2, and the other is bent to form a regular hexagon. What is the area of the hexagon?

   (A) 2 (B) $\frac{3}{2}\sqrt{3}$ (C) 3 (D) $2\sqrt{3}$ (E) 4

5. Dale's lawn, which is circular with a diameter of 30 m, is in need of re-sodding. Dale can only buy sod in 50 cm wide strips. Which of the following best approximates the total length of sod (measured in metres) that Dale needs?

   (A) 500 (B) 900 (C) 1500 (D) 3000 (E) 4500

6. The point P is inside a square whose side length is 16. Its distance from two adjacent vertices and the midpoint of the side opposite these vertices has a common value d. What is the value of d?

   (A) $16\left(2-\sqrt{2}\right)$ (B) 8 (C) $\dfrac{16}{\sqrt{2}}$ (D) $4\left(2+\sqrt{2}\right)$ (E) 10

7. Consider the product

$$\left(0.\overline{9}\right)\left(0.\overline{6}\right)\left(0.24\overline{9}\right)\left(0.1\overline{9}\right)\left(0.8\overline{3}\right).$$

   The line over a digit means that the digit is repeated indefinitely. For example,

$$0.\overline{3} = 0.33333333\cdots = \frac{1}{3} \text{ and } 0.1\overline{6} = 0.16666666\cdots = \frac{1}{6}.$$

   Which of the following is equal to the product above?

   (A) $\dfrac{5}{24}$ (B) $\dfrac{1}{24}$ (C) $\dfrac{1}{30}$ (D) $\dfrac{1}{36}$ (E) $\dfrac{1}{48}$

8. Find the sum of all values x such that $(x^2 - 7x + 11)^{x^2+3x-10} = 1$.

   (A) 0 (B) 2 (C) 4 (D) 7 (E) 9

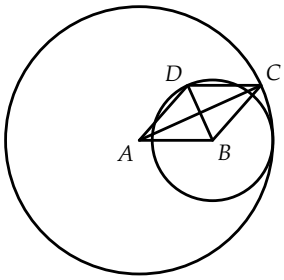
9. Let x be a real number between 0 and 1. When a discount x is applied to the price P of an item in a store, the price of the item is reduced by xP dollars. Three successive discounts of $(\frac{1}{3}x)$ results in a price that is the same as if there were one discount of:

(A) x (B) $x - \frac{x^2}{3} + \frac{x^3}{27}$ (C) $x + \frac{x^2}{3} - \frac{x^3}{27}$ (D) $x - \frac{x^3}{27}$ (E) $x + \frac{x^3}{27}$

10. Nine people attend a dinner where there are three choices for the type of meal. Three people order Combo A, three order Combo B, and three order Combo C. The server distributes the nine meals in random order. In how many different ways can exactly one person receive the correct meal?

(A) 54 (B) 135 (C) 162 (D) 216 (E) 270

11. A rhombus, $ABCD$, has sides of length 1. A circle with centre A passes through C (the opposite vertex.) Likewise, a circle with centre B passes through D. If the two circles are tangent to each other, find the area of the rhombus.



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{8}$

(D) $\frac{3}{4}$ (E) 1

12. For $n = 1, 2, 3, \ldots$, the n^{th} *harmonic number* H_n is defined by

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

Which of the following is a formula for the sum $H_1 + H_2 + \cdots + H_n$?

(A) $(n+1)\,H_n - n$ (B) $(n-1)\,(H_n + 1)$ (C) $nH_n + \frac{1}{n}$

(D) $(n+1)\,H_n + n - 2$ (E) $(2n-1)\,H_n - n$