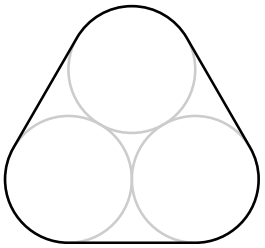
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2017

Senior Final, Part B – Draft 7

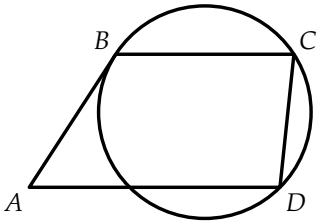
Friday, May 5

key:17012 1. Three cylindrical pipes of radius 1 m are held together by a tight metal band as shown. The length of the metal band is $a + b\pi$ meters, where a and b are positive integers. Determine a and b.



key:16018 2. Six red balls and k blue balls are randomly placed in a line. The probability that the balls at each end of the line are the same color is $\frac{1}{2}$. Find all possible values of k.

key:17039 3. A circle passes through the vertices B, C and D of trapezoid $ABCD$. If AB is a tangent to this circle, Prove that $BD = \sqrt{AD \times BC}$.



key:17050c 4. The function f is not defined at $x = 0$, but for all non-zero real numbers x the function f satisfies the equation

$$2f(x) + f\left(\frac{1}{x}\right) = 15x.$$

Determine the smallest integer n for which $f(n) > 2017$.

key:17031b 5. Let a, b, c be real numbers for which $a + b + c = 0$, $a^2 + b^2 + c^2 = 30$, and $a^3 + b^3 + c^3 = 60$. Determine the values of $a^4 + b^4 + c^4$ and $a^5 + b^5 + c^5$.