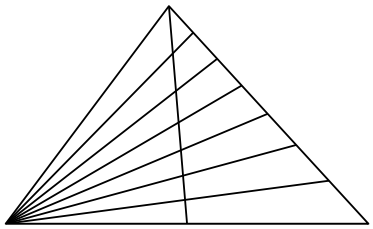
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2016

Senior Final, Part B

Friday, May 6

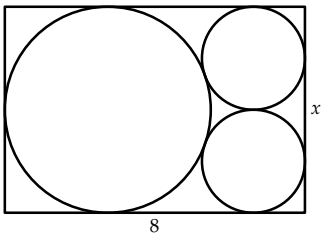
1. How many triangles appear in the diagram?



2. Kelly has a pear orchard. Over the last several years Kelly has found that each tree produces the same number of pears per year as there are pear trees in the orchard. Kelly divides the pears produced during each year among her seven daughters, and she keeps any pears that are left over for herself. Show that Kelly can never end up with 3 pears.

3. Show that $n^5 - 5n^3 + 4n$ is divisible by 40 for all positive integers n.

4. A rectangle has width 8 and height x. Three circles inscribed in the rectangle are tangent to each other and to the rectangle, as shown. Determine x.



5. Find all triplets (p, q, r) where p, q, and r are positive integers of which at least two are prime, for which

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{r}.$$