BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2017

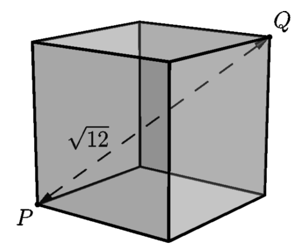
Senior Final, Part A – Draft 7

Friday, May 5

key:17062 1. Let x, y, z be real numbers such that $x + y - z = 2$, $x - y + z = 4$, and $-x + y + z = 6$. Determine the value of $x + y + z$.

(A) 6 (B) 8 (C) 10 (D*) 12 (E) 24

key:17009 2. A cube has diagonal PQ with length $\sqrt{12}$ as shown. Determine the volume of the cube.



(A*) 8 (B) 12 (C) $12\sqrt{2}$

(D) 27 (E) $12\sqrt{2}$

key:17083 3. Alice is driving to Bob's house, intending to arrive at a certain time. If she drives at 60 km/h she will arrive 5 minutes late. If she drives at 90 km/h she will arrive 5 minutes early. If she drives at x km/h she will arrive exactly on time. Determine x.

(A) 66 (B) 70 (C*) 72 (D) 75 (E) 78

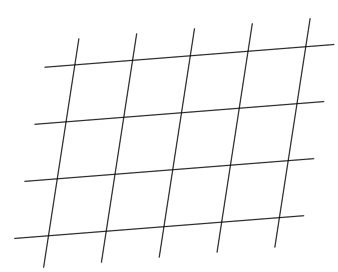
key:17074b 4. Anya and Bert play a game where they flip a coin that is equally likely to come up heads or tails. They take turns flipping the coin, with Anya going first. This first person to flip tails wins. Determine the probability that Anya wins the game.

(A) $\frac{1}{2}$ (B*) $\frac{2}{3}$ (C) $\frac{3}{5}$ (D) $\frac{3}{4}$ (E) $\frac{5}{6}$

key:17035 5. The area of $\triangle ABC$ is 1. Points M, K and P are on the segments AB, BC and CA, respectively, so that $AM = \frac{1}{5}AB$, $BK = \frac{1}{3}BC$, and $CP = \frac{1}{4}CA$. The area of $\triangle MKP$ is

(A*) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D) $\frac{11}{15}$ (E) $\frac{13}{20}$

key:17043 6. Five parallel lines are drawn, and then four other parallel lines are drawn in a different direction. How many distinct parallelograms are there in the picture?



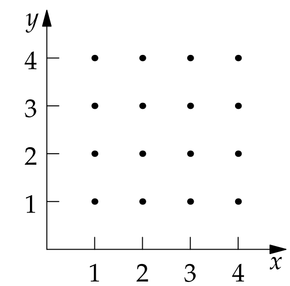
(A) 30 (B) 45 (C) 52 (D*) 60 (E) 100

key:15018 7. Let $|a|$ be the absolute value of the number a. The points (x, y) on the coordinate plane satisfying $|x| \leq 2$, $|y| \leq 2$, and $\big||x| - |y|\big| \leq 1$ define a region with area:

(A) 8 (B) 10 (C*) 12 (D) 14 (E) 16

key:17045b 8. In the xy-plane, consider the sixteen points (x, y) with x and y both integers such that $1 \leq x \leq 4$ and $1 \leq y \leq 4$ (as shown in the diagram). Determine the number of ways we can label ten of these points $A, B, C, D, E, F, G, H, I, J$ such that the nine distances AB, BC, CD, DE, EF, FG, GH, HI, IJ satisfy the inequality

$$AB < BC < CD < DE < EF < FG < GH < HI < IJ.$$



(A) 4 (B) 8 (C) 12 (D*) 24 (E) 36

key:17021 9. There are two integers n such that $\frac{n^2-71}{7n+55}$ is a natural number. The sum of these two integers is

(A) -21 (B) 13 (C) 32 (D*) 49 (E) 98

key:17091 10. Let ABC be an acute-angled triangle with $\cos A = 1/50$. The point O is the centre of the circumcircle of triangle ABC, and I is the centre of the incircle of triangle ABC. Determine the maximum possible value of AI/AO.

(A*) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$