BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2017

Junior Preliminary

Wednesday, April 5

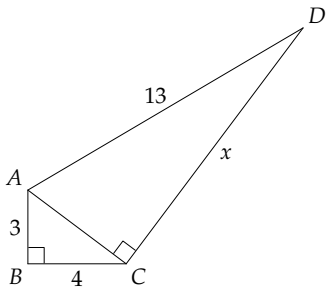
1. If x is a number larger than 5, which of the following expressions is the smallest?

   (A) $5/(x-1)$ (B) $5/x$ (C) $5/(x+1)$ (D) $x/5$ (E) $(x+1)/5$

2. If $3^x + 3^x + 3^x = 243$, determine the value of x.

   (A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) 3 (D) 4 (E) 5

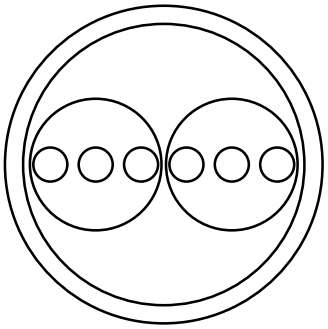
3. In the figure shown, $ABCD$ is a quadrilateral with $\angle ABC = 90^\circ$ and $\angle ACD = 90^\circ$. If $AB = 3$, $BC = 4$, $AD = 13$ and $CD = x$, determine x.

   (A) 5 (B) 10 (C) 12

   (D) $\sqrt{194}$ (E) none of these

4. There are 10 circles in the diagram shown. How many ways are there to label three of the circles, with labels A, B, and C, so that circle A is inside circle B, which is inside circle C?

   (A) 6 (B) 8 (C) 12

   (D) 14 (E) 20

5. All of the edge lengths of a cube are doubled. By what percentage does the volume of the cube increase?

   (A) 100 (B) 200 (C) 300 (D) 400 (E) 700

6. A rectangular box has six faces, whose areas are 5, 5, 10, 10, 18 and 18 cm^2. Determine the volume of this box.

   (A) 15 cm^3 (B) 30 cm^3 (C) 45 cm^3 (D) 60 cm^3 (E) 90 cm^3

7. The average age of 120 people is 35. The average age of the men is 32, while the average age of the women is 37. How many women are there?

   (A) 60 (B) 66 (C) 72 (D) 75 (E) 80

8. Four students each roll a 6-sided die (assume the numbers 1 through 6 are equally likely to occur). What is the probability that they all roll different numbers?

   (A) less than 0.15 (B) between 0.15 and 0.3 (C) between 0.3 and 0.45

   (D) between 0.45 and 0.6 (E) greater than 0.6

9. A group of strangers attended a party. Each person shook hands with everyone else. Unfortunately, Chris arrived late and was able to shake hands with only some of the guests. If there were a total of 25 handshakes (including all of Chris's), how many hands did Chris shake?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

10. Recall that $n! = n \cdot (n-1)(n-2) \cdots 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Which of the following numbers is a perfect square?

    (A) $\frac{4!5!}{2}$ (B) $\frac{5!6!}{2}$ (C) $\frac{6!7!}{2}$ (D) $\frac{7!8!}{2}$ (E) $\frac{8!9!}{2}$

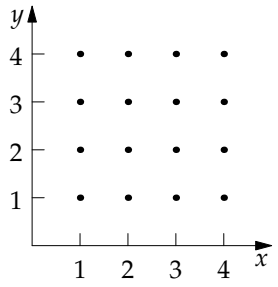
11. Let $S = \{a, b, c, d, e\}$ be a set of positive integers with $a < b < c < d < e$. When two different numbers are chosen from this set, the possible values for their sum are:

    $$165, 170, 175, 177, 182, 187, 190, 195, 200, 207.$$

    Determine the value of c.

    (A) 90 (B) 91 (C) 92 (D) 93 (E) 94

12. In the xy-plane, consider the sixteen points (x, y) with x and y both integers such that $1 \leq x \leq 4$ and $1 \leq y \leq 4$ (as shown in the diagram). Determine the number of triangles with positive area whose three vertices are chosen from these sixteen points.



    (A) 496 (B) 516 (C) 520

    (D) 528 (E) 560