BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2018

Junior Preliminary Problems & Solutions

1. If 6 is the average of the four numbers $3, 4, 5$, and x, then the value of x is:

   (A) 7 (B) 9 (C) 10 (D) 12 (E) 18

   Solution
   Average $=\frac{sum\ of\ values}{number\ of\ values}$. So $\frac{3+4+5+x}{4} = 6$. Thus $12 + x = 24, x = 12$.

   Answer: D

2. A one pound bag of mixed nuts contains 60% peanuts, and a three-pound bag contains 40% peanuts. If these bags are mixed together to obtain a four-pound bag of nuts, then the percentage of peanuts in the bag is:

   (A) 42 (B) 42.5 (C) 45 (D) 50 (E) 55

   Solution
   One pound with 60% peanuts contains 0.6 pounds total, and 3 pounds containing 40% peanuts have 1.2 pounds total. Therefore

   $$\frac{Total\ peanuts}{Total\ nuts} = \frac{1.2 + 1.6}{1 = 3} = \frac{1.8}{4} = 0.45,\ or\ 45\%.$$

   Answer: C

3. We say a positive integer is “happy" if it is less than 100 and is divisible by either 3 or 7, or both. For example, $3, 70$ and 84 are all happy. The number of happy numbers is:

   (A) 39 (B) 40 (C) 43 (D) 45 (E) 47

   Solution
   The number of values less than 100 and divisible by 3 equals 33. The number of values less than 100 and divisible by 7 equals 14. The number of values divisible by both is 4. We have $33 + 14 - 4 = 43$.

   Answer: C

4. Andre, Bert, Curtis and David are playing on a see-saw. Andre is heavier than Bert and Curtis together. Andre and Bert together balance perfectly with Curtis and David together. Bert and David together are heavier than Andre and Curtis together. Listed from lightest to heaviest, the order of the boys is:

   (A) Bert, Curtis, David, Andre (B) Curtis, Bert, Andre, David (C) Curtis, Bert, David, Andre

   (D) David, Bert, Curtis, Andre (E) David, Curtis, Bert, Andre

   Solution

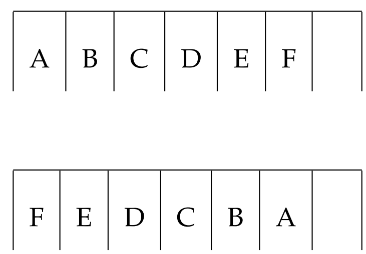
We have

$$\begin{aligned} (1)\quad & A && > B + C \\ (2)\quad & A + B && = C + D \\ (3)\quad & B + D && > A + C \end{aligned} \cdot$$

Adding (2) + (3), we get $A + 2B + D > A + 2C + D$. So $B > C$, and hence $A > B > C$. By (2) and $B > C$, we get $D > A$. Thus $C < B < A < D$.

Answer: B

5. Six cars, labelled A, B, C, D, E, and F, are parked adjacent to one another in six of seven adjacent parking spaces, as shown in the top picture to the right. If a "move" means moving a car to an empty space (not necessarily an adjacent space), then the smallest number of moves needed to put the cars in reverse order, as shown in the lower picture, is:



(A) 7 (B) 8 (C) 9

(D) 10 (E) 12

Solution

It takes 3 moves to switch a pair such as A and F (e.g. move A to empty spot, move F to A's old spot, move A to F's old spot). There are 3 pairs that need to switch positions: A-F, B-E, C-D. $3 \times 3 = 9$.

Answer: C

6. If four fair coins are flipped, then the probability of at least two coins coming up "heads" is:

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$ (E) $\frac{11}{16}$

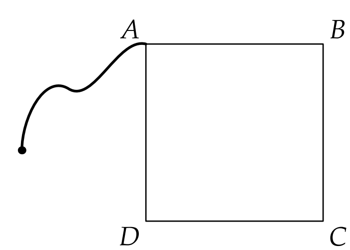
Solution

Four coins have 16 possibilities: HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, $HTTT$, THHH, THHT, THTH, $THTT$, TTHH, $TTHT, TTTH, TTTT$.

The probability is $\frac{11}{16}$

Answer: E

7. A cow is tied with a rope to the corner of the square shed $ABCD$, as shown. If the length of the rope is 6 and the length of each side of the shed is 4, then the area the cow can graze is:



(A) 28π (B) 29π (C) $36\pi - 16$

(D) $40\pi - 16$ (E) 36π

Solution

The area equals $\frac{3}{4}$ area of a circle of radius 6 plus $\frac{1}{2}$ area of circle of radius 2. Thus

$$Area = \frac{3}{4}\pi 6^2 + \frac{1}{2}\pi 2^2 = 27\pi + 2\pi = 29\pi.$$

Answer: B

8. A straight line passes through three points with coordinates $(0, 12)$, $(x, 93)$ and $(100, 120)$. The value of x is:

(A) 69 (B) 70 (C) 72 (D) 75 (E) 77

Solution

$$slope = \frac{120 - 12}{100 - 0} = \frac{108}{100} = \frac{93 - 12}{x}.$$

Therefore, $108x = 8100$, $x = \frac{8100}{108} = 75$.

Answer: D

9. If a quadrilateral is drawn with vertices at the points $(0, 3)$, $(2, 1)$, $(6, 3)$ and $(4, 6)$, then its area in square units is:

(A) 12 (B) 15 (C) 16 (D) 18 (E) 19

Solution
For the triangle $\triangle ABC$ with points $A = (0, 3), B = (6, 3), C = (4, 6)$, the vertical distance (i.e. height of $\triangle ABC$) from the line from $\overline{AB}$ to the point C equals 3 units. Thus the area of $\triangle ABC$ equals one half the length of $\overline{AB}$ times its height equals $\frac{1}{2} \cdot 6 \cdot 3 = 9$. The height of $\triangle ABD$ (where $D = (2, 1)$) as measured from $\overline{AB}$ to D equals, 2 units. Thus the area of $\triangle ABD$ equals one half the length of $\overline{AB}$ times the height of $\triangle ABD = \frac{1}{2} \cdot 6 \cdot 2 = 6$. The area of the quadrilateral is therefore $9 + 6 = 15$ units.

Answer: B

10. Joshua thinks of a 3-digit number, and you are supposed to determine the number by asking questions. The only type of question you may ask is one that has an answer of "yes" or "no". The minimum number of questions in a strategy that is sure to determine the number (because it does not rely on luck) is:

(A) 8 or fewer (B) 9 to 11 (C) 12 to 19 (D) 20 to 100 (E) more than 100

Solution
The three digits go from 100 to 999, 899 numbers in total. You can rule out half each time with questions like "Is it below 550?". It it's true, your next question would be: Is it below 325? Otherwise, if it's above 550, your next question would be, "is it below 775?". Each time one reduces the list of possible numbers by one half. Now since

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 900 = \frac{900}{1024} < 1,$$

one will need at most 10 questions to determine the right number.

Answer: B

11. The remainder when $1^{2018} + 3^{2018} + 5^{2018} + 7^{2018} + 9^{2018}$ is divided by 20 is:

(A) 5 (B) 7 (C) 13 (D) 15 (E) 17

Solution

Obviously, when divided by 20, the remainder for any power of 1 equals 1. The powers of 3: $3, 9, 27, 81, 243, 729, \ldots$ when divided by 20 have remainders following the pattern $3, 9, 7, 1, 3, 9, 7, 1 \ldots$. Since $2018 \div 4$ has a remainder of 2. So $3^{2018} \div 20$ has a remainder which is second in the pattern, that is, 9. $5^n \div 20$ always has a remainder of 5. $7^n \div 20$ has remainder pattern $7, 9, 3, 1, 7, 9, 3, 1, \ldots$ and $9^n \div 20$ has remainder pattern $9, 1, 9, 1, \ldots$. So $7^{2018} \div 20$ has remainder 9 and $9^{2018} \div 20$ has remainder 1. Adding these remainders, $1 + 9 + 5 + 9 + 1 = 25$. Given that $25 \div 20$ has a remainder of 5, the answer is 5.

Answer: A

12. Each of the letters $A, B, C, D, E, F, G, H, I$, and J is assigned to exactly one digit from 0 through 9, with no two letters being assigned to the same digit. The digits have the property that $ABABFJAI \div ABC = DEFGD$, with no remainder. To the right is the completed long division. The digit assigned to H is:

(A) 5 (B) 6 (C) 7

(D) 8 (E) 9

```
            D E F G D
       ________________
ABC )  A B A B F J A I
 2     A A A I
       ________
 3         D I F
           F I F
           ______
 1         A G G J
             D D B
             _____
               A A A
               G G G
               _____
               A A A I
               A A A I
```

Solution

From 1 we see that $A = 1$. By 3, it follows that $B = 2$. Looking at 2 again, we have $10 - A = D$, and hence $D = 9$. Also, $12 - I = I$. Therefore, $I = 6$. Looking at 3, again, we have $9 - F = A = 1$. Therefore, $F = 8$. Also, by 1, it follows that $G = 0$. Going back to 1, $D \times C = I (mod\ 10)$ or $9C = 6 (mod\ 10)$. Thus $C = 4$. From 1, we have $J - B = A$. So $J = 2 + 1 = 3$. From 3, we have $E \times C = F (mod\ 10)$, or $4E = 8 (mod\ 10)$. Given that $E \neq 2$, it follows that $E = 7$. We have: $A = 1, B = 2, C = 4, D = 9, E = 7, F = 8, G = 0, H = ?, I = 6, J = 3$. It follows that $H = 5$, the last digit available.

Answer: A