# BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2016

## Junior Final, Part A Problems & Solutions

1. Theo got some birthday money from his grandparents. He spent half the money on a video game and a third of what was left on Pokémon cards. He now has $40 left. How much money did he get?

   (A) $80 (B) $120 (C) $160 (D) $200 (E) $240

   **Solution**

   Theo spent half of his money on the video game, and a third of the remaining half on Pokemon. He now has two thirds of one half left. Two thirds of one half is one third of his birthday money, and if one third of the whole amount is $40, then the whole amount is three times that, or $120.

   **Answer: B**

2. Consider the product $54m$, where $m$ is a positive integer. The smallest possible value of $m$ for which the product is a perfect square is:

   (A) 4 (B) 6 (C) 9 (D) 12 (E) 24

   **Solution**

   In a perfect square, every prime factor occurs an even number of times. Note that $54 = 2 \times 3^3$. Thus, if the product $54m$ is to be a perfect square, then the smallest possible value of $m$ is $m = 2 \times 3 = 6$.

   **Answer: B**

3. How many 3-digit odd numbers use each of the digits 0, 3, and 5 exactly once?

   (A) 2 (B) 6 (C) 4 (D) 5 (E) None of these

   **Solution**

   If our number has three digits, then it can't start with 0, and if it is odd, then it can't end with 0 either. If each of the digits 0,3, and 5 need to each be used exactly once, then our only choices are 305 and 503. Answer: 2.

   **Answer: A**

4. Ten people decide to buy a car together, sharing the cost equally. If there had been five more in the group, the cost to each person would have been $100 less. What is the cost of the car?

   (A) $1500 (B) $3000 (C) $5000 (D) $6000 (E) $15000

   **Solution**

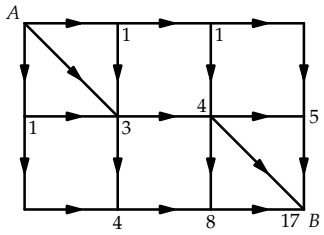
   Suppose each person pays $x$ dollars when there are 10 people. Then the total cost is $10x$. For fifteen people, each person pays $x - 100$, so the total cost is $15(x - 100)$. Setting the two expressions for total cost equal to each other, we get

   $$10x = 15x - 1500 \implies 5x = 1500 \implies 10x = 3000 = \text{total cost.}$$

   Answer: $3000.

   **Answer: B**

5. Travelling only on paths that follow the arrows, how many paths are there from $A$ to $B$?




(A) 10 (B) 13 (C) 17

(D) 24 (E) 72

**Solution**

Put a number at each vertex signifying how many paths there are from $A$ to that point. To do this, start at $A$ and work to the right and down: at each vertex, write the sum of the numbers at the vertices that "feed" into that vertex. See diagram! Answer: $4 + 5 + 8 = 17$.

**Answer: C**

6. The number of positive integers $n$ for which $\dfrac{n}{30-n}$ is a positive integer is:

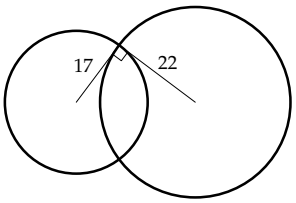
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

**Solution**

For the given expression to be a positive integer, we must have $n \geq 30 - n > 0$ and also $30 - n$ must be a factor of $n$. The two inequalities imply $15 \leq n < 30$. Successively trying all $n$ in the interval, we find there are seven values for which the given expression is a positive integer; namely, 15, 20, 24, 25, 27, 28, and 29.

**Answer: D**

7. A circle of radius 17 intersects another circle, radius 22, at right angles as shown. What is the difference of the areas of the non-overlapping portions?




(A) $115\pi$ (B) $135\pi$ (C) $155\pi$

(D) $175\pi$ (E) $195\pi$

**Solution**

The area of the larger circle is $22^2\pi$; the area of the smaller circle is $17^2\pi$. Suppose $x$ is the area of the overlapping portion. Then the difference of the areas of the non-overlapping portions is

$$\begin{aligned}(22^2\pi - x) - (17^2\pi - x) &= (22^2 - 17^2)\pi \\ &= (22 + 17)(22 - 17)\pi \\ &= (39)(5)\pi = 195\pi.\end{aligned}$$

**Answer: E**

8. In the addition shown below, $A$, $B$, $C$, and $D$ are distinct digits. The number of different possible values for $D$ is:

$$\begin{array}{cccccc} & & A & E & C & A \\ + & & B & C & D & B \\ \hline & F & E & E & D & D \end{array}$$

(A) 2 (B) 4 (C) 6 (D) 8 (E) 9

**Solution**

We know $A + B > 9$ because $F$ is a "carried" digit. Therefore, since $1 + C + D = D + 10$, we know that $C$ must be 9. Since $A$, $B$ and $C$ are distinct, $A$ and $B$ cannot be 9. The possible two-digit sums of distinct digits are $10(= 4 + 6)$, $11(= 5 + 6)$, $12(= 5 + 7)$, $13(= 6 + 7)$, $14(= 6 + 8)$ and $15(= 7 + 8)$. Answer: 6.

**Answer: C**

9. A room has a floor that is 6 m wide and 9 m long. The ceiling of the room is 3 m high. The room requires soundproof padding on the inner sides of the walls and ceiling, but not on the floor. The padding is 0.5 m thick. The total volume of padding required, measured in cubic metres, is:

(A) 62 (B) 64 (C) 68.5 (D) 70 (E) 72

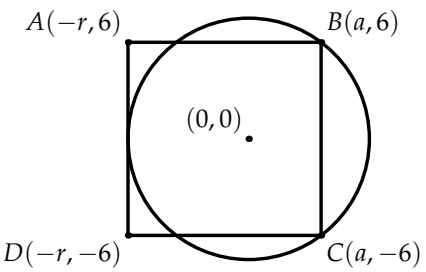
**Solution**

A 6 m wide and 9 m sheet is required for the ceiling. Once this is installed, the walls are 2.5 m high. Then two 2.5 m $\times$ 9 m sheets are required for the two long walls. Once these are installed, the two short walls are 5 m $\times$ 2.5 m. Therefore, the total volume of the padding is

$$\text{volume of padding} = 6 \times 9 \times 0.5 + 2\,(2.5 \times 9 \times 0.5) + 2\,(2.5 \times 5 \times 0.5) = 0.5\,(6 \times 9 + 5 \times 9 + 5 \times 5) = 62$$

**Answer: A**

10. The square $ABCD$ has side length 12. A circle of radius $r$ drawn through $B$ and $C$ is tangent to $AD$. Find $r$.




(A) 6 (B) $\frac{15}{2}$ (C) $3\sqrt{2}$

(D) 8 (E) $6\sqrt{2}$

**Solution**

The equation of the circle is $x^2 + y^2 = r^2$. Vertex $B(a, 6)$ is on the circle so $a^2 + 6^2 = r^2$. Also, side $AB$ has length $r + a = 12$. We have two equations in two unknowns:

$$\begin{cases} a^2 + 36 = r^2 \\ r + a = 12 \end{cases} \implies (12 - r)^2 + 36 = r^2 \implies r = 15/2$$

**Answer: B**