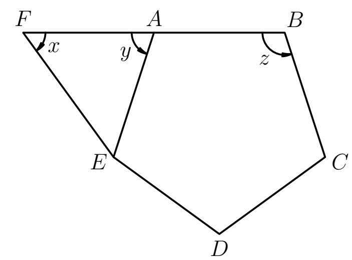
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2015

Senior Final, Part A

Friday, May 1

1. In the diagram, $ABCDE$ is a regular pentagon, and FAB is a straight line. Further, $FA = AB$. For the angles x, y, and z the ratio $x : y : z$ is:



   (A) 1 : 2 : 3 (B) 2 : 2 : 3 (C) 2 : 3 : 4

   (D) 3 : 4 : 5 (E) 3 : 4 : 6

2. Recall that $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. Only one of the expressions below is a perfect square. It is:

   (A) $\dfrac{23! \cdot 24!}{3}$ (B) $\dfrac{24! \cdot 25!}{3}$ (C) $\dfrac{25! \cdot 26!}{3}$ (D) $\dfrac{26! \cdot 27!}{3}$ (E) $\dfrac{27! \cdot 28!}{3}$

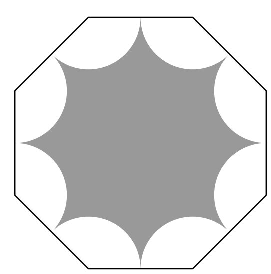
3. Al and Bev play a game where they take turns removing 1, 2, 3, 4, or 5 coins from a pile which originally contains 2015 coins. The player who takes the last coin wins the game. If Al goes first, there is a strategy that he can use that prevents Bev from winning. Using this strategy the number of coins he should take on his first move in order to prevent Bev from winning is:

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. The six sides of a rectangular box have a total surface area of 175 square centimetres. The sum of the lengths of all twelve edges of the box is 80 centimetres. The sum of the lengths, measured in centimetres, of the four interior diagonals of the box is:

   (A) $16\sqrt{5}$ (B) $20\sqrt{7}$ (C) 60 (D) 75 (E) $30\sqrt{3}$

5. A regular octagon has side length 6. Identical arcs with radius 3 are drawn with centre at each of the vertices of the octagon, creating circular sectors as shown. The region inside the octagon but outside of any of the sectors is shaded. The area of the shaded region is:



   (A) $9\left(8 + 8\sqrt{2} - 3\pi\right)$ (B) $9\left(16 - 3\pi\right)$ (C) $24\left(6 - \pi\right)$

   (D) $8\left(9 + 9\sqrt{2} - 3\pi\right)$ (E) $9\left(12 + 8\sqrt{2} - 3\pi\right)$

6. The sum of the digits of the number 28531 is 19. If N is the smallest positive integer with a sum of digits of 2015, the sum of the digits in $N + 1$ is:

   (A) 1 (B) 9 (C) 223 (D) 224 (E) 2016

7. If $p = 1\log_{10} 1 + 2\log_{10} 2 + 3\log_{10} 3 + 4\log_{10} 4 + 5\log_{10} 5 + 6\log_{10} 6$, then the number 10^p is an integer. The largest power of 2 that is a factor of 10^p is:

(A) 2^{12} (B) 2^{14} (C) 2^{16} (D) 2^{18} (E) 2^{20}

8. The numbers 1, 2, 3, 4, and 5 are to be arranged in a circle. An arrangement is *good* if it is true that for every n from 1 to 15 it is possible to find a subset of the numbers that appear consecutively on the circle that sum to n. Otherwise the arrangement is *bad*. For example, the arrangement 1, 3, 5, 4, 2 is good, since:

$$1 = 1,\ 2 = 2,\ 3 = 3,\ 4 = 4,\ 5 = 5,\ 6 = 4 + 2 = 2 + 1 + 3,\ 7 = 4 + 2 + 1$$

$$8 = 3 + 5,\ 9 = 5 + 4,\ 10 = 4 + 2 + 1 + 3,\ 11 = 5 + 4 + 2,\ 12 = 5 + 4 + 2 + 1$$

$$13 = 1 + 3 + 5 + 4,\ 14 = 3 + 5 + 4 + 2,\ 15 = 1 + 3 + 5 + 4 + 2$$

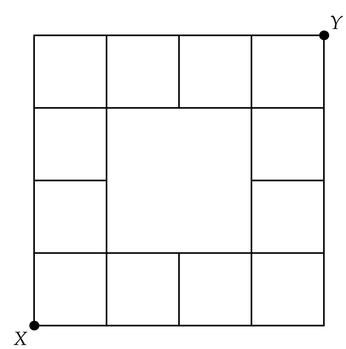
Arrangements that differ only by any combination of rotations or reflections are to be considered the same. The number of different *bad* arrangements is:

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

9. In the 4×4 square shown, it is required to get from X to Y moving along only the black lines. The total number of shortest routes from X to Y is:

(A) 18 (B) 26 (C) 32

(D) 34 (E) 36



10. The number of positive integers strictly less than 2015 that are divisible by exactly two of 5, 7, and 9 is:

(A) 102 (B) 114 (C) 126 (D) 120 (E) 132

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Senior Final, Part B

Friday, May 1

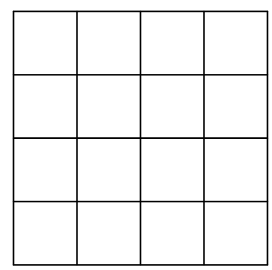
1. A helicopter flies north for some time at 80 km/hr, then flies 10 km east at 50 km/hr, and then flies directly back to the starting point at 80 km/hr. If whole trip took one hour, how far north did the helicopter fly during the first leg of the trip?

2. Hayden has a lock with a combination consisting of two 8s separated by eight digits, two 7s separated by seven digits, two 6s separated by six digits, all the way down to two 1s separated by one digit. For example, two 1s are separated by one digit in ___ 121 ___. Unfortunately, Hayden spilled coffee on the paper that the combination was written on, and all that can be read of the combination is:

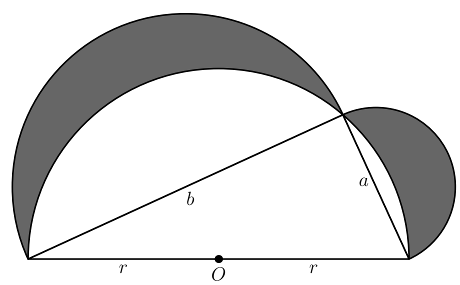
   _ _ _ 5 8 4 _ _ _ _ _ _ _ _ _ _

   Determine one of the two possible combinations of the lock.

3. Consider the 4×4 lattice shown. Determine the number of rectangles with horizontal and vertical sides that can be formed using four of the intersection points in the lattice as vertices. Note that a square is a rectangle.



4. Inscribe a triangle with side lengths $a \leq b < 2r$ inside a semi-circle centred at O with radius r. On each of the two shorter sides of the triangle construct two smaller semi-circles with diameters of lengths a and b, respectively. Prove that the sum of the areas of the two lunar regions, shown shaded in the diagram, between the three semi-circles equals the area of the triangle.



5. Let P be the parabola with equation $y = x^2$ and let $Q = (20, 14)$. There are real numbers r and s such that the line through Q with slope m does not intersect P if and only if $r < m < s$. Determine the value of $r + s$.