BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2015

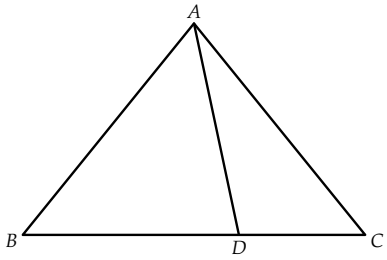
Junior Final, Part A

Friday, May 1

1. A two-digit number is divisible by 8, 12, and 18. The number is between:

   (A) 10 and 19 (B) 20 and 39 (C) 40 and 59 (D) 60 and 79 (E) 80 and 99

2. Triangle ABC is isosceles with $AB = AC$. Further, $AD = BD$ and $\angle DAC = 27°$. The value of $\angle BDA$, measured in degrees, is:



   (A) 102 (B) 78 (C) 60

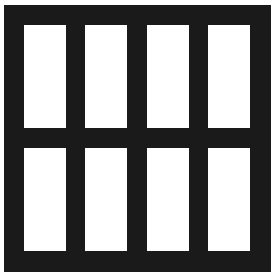
   (D) 51 (E) 27

3. A lamp is red for 1 second, then blue for 2 seconds, then green for 3 seconds, then red for 1 second, then blue for 2 seconds, then green for 3 seconds, and so on. Precisely 2015 seconds after this process starts, the lamp is:

   (A) green (B) blue (C) changing from red to blue

   (D) changing from blue to green (E) changing from green to red

4. Dana constructs a square window with side length s using 8 equal-size panes of glass as shown in the diagram. The ratio of the height to width of each pane is 5 : 2, and the borders around and between the panes are 5 cm wide. The value of s, measured in centimetres, is:



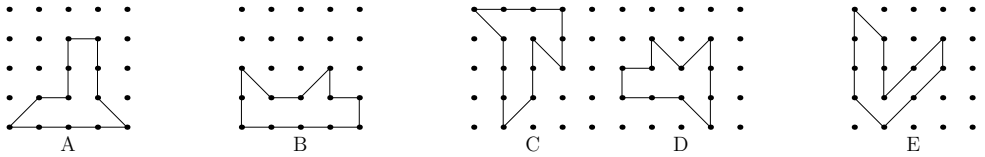
   (A) 70 (B) 65 (C) 60

   (D) 55 (E) 48

5. Cary's cat eats $\frac{1}{3}$ of a can of cat food every morning and $\frac{1}{4}$ of a can every evening. Before feeding the cat on Monday morning, Cary opened a box containing 6 cans of cat food. On the day that the cat finished eating all of the cat food in the box, there was not enough food left to feed it for the day. So Cary opened another can of cat food. The day of the week when the extra can of cat food was emptied was:

   (A) Tuesday (B) Wednesday (C) Thursday (D) Friday (E) Saturday

6. In the diagram below, the polygon with the largest area is:



(A) A (B) B (C) C (D) D (E) E

7. A suspension bridge connects points A and D, which are 60 m apart at equal elevation on each side of a canyon. Segment BC is a rigid level platform 36 m long centred in the canyon. A cable 76 m long connects A to B to C to D. If the cable is shortened by 10 m, while the platform remains level and centred in the canyon, the amount the platform will rise, measured in metres, is:

(A) 5 (B) 7 (C) 9 (D) 10 (E) 12

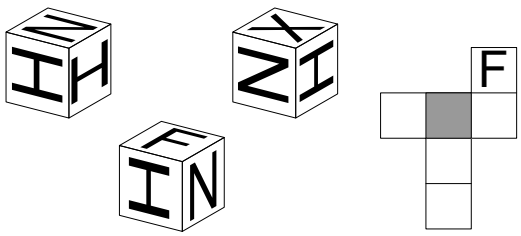
8. Turbo the tortoise goes 1 kilometre uphill at 2 kilometres per hour, 2 kilometres on level ground at 3 kilometres per hour, and 3 kilometres downhill at 4 kilometres per hour. Turbo's average speed, measured in kilometres per hour, for the whole journey is:

(A) 2 (B) $2\frac{1}{2}$ (C) 3 (D) $3\frac{1}{3}$ (E) $3\frac{3}{23}$

9. Recall that $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. Only one of the expressions below is a perfect square. It is:

(A) $\dfrac{23! \cdot 24!}{3}$ (B) $\dfrac{24! \cdot 25!}{3}$ (C) $\dfrac{25! \cdot 26!}{3}$ (D) $\dfrac{26! \cdot 27!}{3}$ (E) $\dfrac{27! \cdot 28!}{3}$

10. The six faces of a cube are labeled F, H, I, N, X, and Z. Three views of the labeled cube are shown. Note that the H and N on the die are indistinguishable from the rotated I and Z, respectively. The cube is then unfolded to form the lattice shown, with the F shown upright. The letter that should be drawn upright in the shaded square is:



(A) H (B) I (C) N

(D) X (E) Z

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Junior Final, Part B

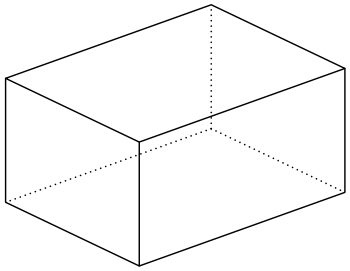
Friday, May 1

1. An arithmetic sequence is an ordered set of terms for which the difference between consecutive terms is a fixed amount. (E.g. 13, 25, 37, 49, 61.) Given that the four three-digit numbers

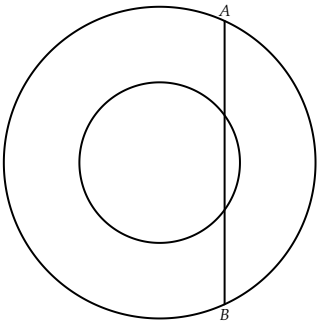
$$abc,\ ba8,\ 6c5,\ 80a$$

where a, b, and c are each one of the digits $0 \ldots 9$, form an arithmetic sequence, determine the digits a, b, and c.

2. A rectangular prism is a solid with six rectangular faces (see the diagram). The edges of a particular rectangular prism have integer lengths, measured in centimetres, with the longest side measuring 139 cm. If the total surface area of the prism is 2530 cm^2, determine the volume of the prism.



3. In the diagram, the two concentric circles are such that the 30 cm chord, AB, of the larger circle is trisected by the smaller circle. If the sum of the radii of the two circles is 25 cm, find the radii of each of the two circles.

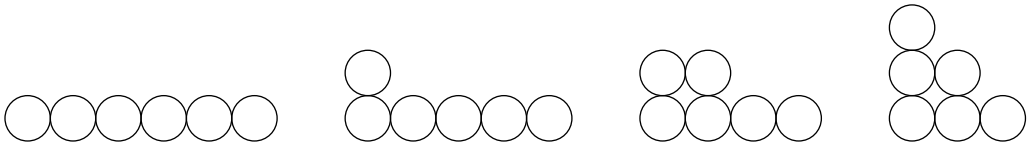


4. Hayden has a lock with a combination consisting of two 8s separated by eight digits, two 7s separated by seven digits, two 6s separated by six digits, all the way down to two 1s separated by one digit. For example, two 1s are separated by one digit in __ 121 __. Unfortunately, Hayden spilled coffee on the paper that the combination was written on, and all that can be read of the combination is:

$$\_\ \_\ \_\ 5\ 8\ 4\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$$

Determine one of the two possible combinations of the lock.

5. Let $A(n)$ represent the number of ways that n pennies can be arranged in any number of rows, where each row starts at the same position as the row below it and has fewer pennies than the row below it. For example, $A(6) = 4$, as shown below:



(a) Show that $A(9) = 8$.

(b) Find, with explanation, the smallest number k which is not equal to $A(n)$ for any n?