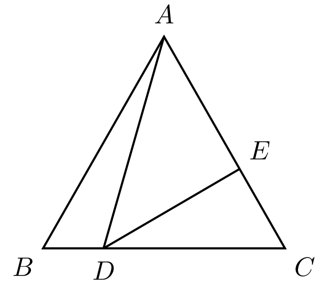
BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2001

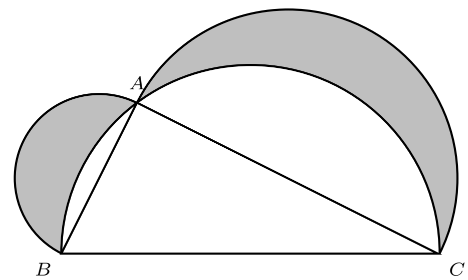
Final Round – Part B

Friday May 4, 2001

1. In the triangle shown $\angle BAD = \alpha$, $\overline{AB} = \overline{AC}$ and $\overline{AD} = \overline{AE}$. Find $\angle CDE$ in terms of α.



2. A semicircle BAC is mounted on the side BC of the triangle ABC. Semicircles are also mounted outwardly on the sides BA and AC, as shown in the diagram. The shaded crescents represent the area inside the smaller semicircles and outside the semicircle BAC. Show that the total shaded area equals the area of the triangle ABC.



3. Five schools competed in the finals of the British Columbia High School Track Meet. They were Cranbrook, Duchess Park, Nanaimo, Okanagan Mission, and Selkirk. The five events in the finals were: the high jump, shot put, 100-metre dash, pole vault and 4-by-100 relay. In each event the school placing first received five points; the one placing second, four points; the one placing third, three points; and so on. Thus, the one placing last received one point. At the end of the competition, the points of each school were totaled, and the totals determined the final ranking.

   i) Cranbrook won with a total of 24 points.
   ii) Sally Sedgwick of Selkirk won the high jump hands down (and feet up), while Sven Sorenson, also of Selkirk, came in third in the pole vault.
   iii) Nanaimo had the same number of points in at least four of the five events.

   Each school had exactly one entry in each event. Assuming there were no ties and the schools ended up being ranked in the same order as the alphabetical order of their names, in what position did Doug Dolan of Duchess Park rank in the high jump.

4. A box contains tickets of two different colours: blue and green. There are 3 blue tickets. If two tickets are to be drawn together at random from the box, the probability that there is one ticket of each colour is exactly $\frac{1}{2}$. How many green tickets are in the box? Give all possible solutions.

5. In (a), (b), and (c) below the symbols m, h, t, and u can represent any integer from 0 to 9 inclusive.

   (a) If $h - t + u$ is divisible by 11, prove that $100h + 10t + u$ is divisible by 11.

   (b) If $h + u = m + t$, prove that $1000m + 100h + 10t + u$ is divisible by 11.

   (c) Is it possible for $1000m + 100h + 10t + u$ to be divisible by 11 if $h + u \neq m + t$? Explain.