BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2001

Final Round – Part A

Friday May 4, 2001

1. The number 2001 can be written as a difference of squares, $x^2 - y^2$ where x and y are positive integers, in four distinct ways. The sum of the four possible x values is:

   (a) 55 (b) 56 (c) 879 (d) 1440 (e) 2880

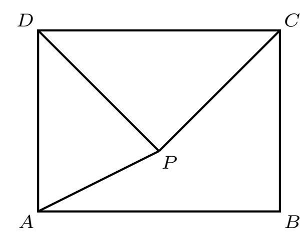
2. Antonino goes to the local fruit stand and spends a total of \$20.01 on peaches and pears. If pears cost 18¢ and peaches cost 33¢, the maximum number of fruits Antonino could have bought is:

   (a) 110 (b) 107 (c) 100 (d) 92 (e) 62

3. The value of $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ is:

   (a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{6}$ (e) 4

4. The point P is interior to the rectangle $ABCD$ such that $\overline{PA} = 3\,\text{cm}$, $\overline{PC} = 5\,\text{cm}$, and $\overline{PD} = 4\,\text{cm}$. Then $\overline{PB}$, in centimeters, is:

   (a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $3\sqrt{3}$
   (d) $4\sqrt{2}$ (e) 2

5. Two overlapping spherical soap bubbles, whose centres are 50 mm apart, have radii of 40 mm and 30 mm. The two spheres intersect in a circle whose diameter, in millimetres, is:

   (a) 36 (b) 48 (c) 50 (d) 54 (e) 64

6. The local theatre charges one dollar for the Sunday afternoon matinee. One Sunday the cashier finds that he has no change. Eight people arrive at the theatre; four have only a one-dollar coin (a loonie) and four have only a two-dollar coin (a toonie). Depending on how the people line up, the cashier may or may not be able to make change for every person in the line as they buy their tickets one at a time. Suppose that the eight people form a line in random order, without knowing who has a loonie and who has a toonie. Then the probability that the cashier will be able to make change for every person in the line is:

   (a) $\dfrac{1}{70}$ (b) $\dfrac{1}{14}$ (c) $\dfrac{1}{7}$ (d) $\dfrac{1}{5}$ (e) $\dfrac{1}{4}$

7. There is a job opening at bcmath.com for a Webmaster. There are three required skills for the position: Writing, Design, and Programming. There are 45 applicants for the position. Of the 45 applicants, 80% have at least one of the required skills. Twenty of the applicants have at least design skills, 25 have at least writing skills, and 21 have at least programming skills. Twelve of the applicants have at least writing and design skills, fourteen have at least writing and programming skills, and eleven have at least design and programming skills. If only those applicants with all three skills will be interviewed, the number of applicants to be interviewed is:

   (a) 3 (b) 7 (c) 8 (d) 9 (e) 11

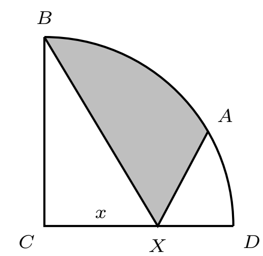
8. Let $a \circledL b$ represent the operation on two numbers a and b, which selects the larger of the two numbers, with $a \circledL a = a$. Let $a \circledS b$ represent the operation which selects the smaller of the two numbers with $a \circledS a = a$. If a, b, and c are distinct numbers, and $a \circledS (b \circledS c) = (a \circledS b) \circledL (a \circledS c)$, then we must have:

(a) $a < b$ and $a < c$ (b) $a > b$ and $a > c$ (c) $c < b < a$
(d) $c < a < b$ (e) $a < b < c$

9. The coordinates of the points A, B, and C are $(7, 4)$, $(3, 1)$, and $(0, k)$, respectively. The minimum value of $\overline{AC} + \overline{BC}$ is obtained when k equals:

(a) 1 (b) 1.7 (c) 1.9 (d) 2.5 (e) 4

10. Given the quarter circle BAD with radius $\overline{BC} = \overline{DC} = 1$, suppose that $\angle BCA = 60°$ and X is a point on segment DC with $\overline{CX} = x$. If the area of the shaded region BXA is one half the area of the quarter circle, then the value of x is:



(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{\pi}{6}$
(d) $\frac{\pi}{4}$ (e) none of these