# BRITISH COLUMBIA COLLEGES

**Senior High School Mathematics Contest**

**Preliminary Round March 10, 1999**

1. One store sold red plums at four for a dollar and yellow plums at three for a dollar. A second store sold red plums at four for a dollar and yellow plums at six for a dollar. You bought $m$ red plums and $n$ yellow plums from each store, spending a total of ten dollars. How many plums in all did you buy?

   (a) 20 (b) 30 (c) 40 (d) 50 (e) not enough information

2. In this unusual games of noughts (Os) and crosses (Xs) the first player to form a line of three Os or three Xs **loses**. It is X's turn. Where should she place her cross to make sure she does not lose?

| | | |
|---|---|---|
| A | O | D |
| B | X | E |
| C | X | O |

   (a) A (b) B (c) C (d) D (e) E

3. Josh found the value of $2^{36} - 1$ to be $68a19476735$ He found all the digits correctly except the third decimal digit which is denoted by $a$. The value of $a$ is:

   (a) 1 (b) 3 (c) 4 (d) 6 (e) 7

4. Given $\log_9 20 = a$ and $\log_3 n = 4a$, the value of $n$ is:

   (a) 400 (b) 100 (c) 80 (d) 20 (e) $\sqrt{20}$

5. The Mathematics Club has a special initiation game. Two candidates for admission are required to wear a party hat with an integer written on it. Each integer must be greater than or equal to zero and at least one of the integers must be nonzero. Each player can see the integer written on the other player's hat, but not on her own. In addition two integers are written on the blackboard, one of which is the sum of the two integers on the players' hats. Every ten seconds the referee of the game rings a bell. When the bell rings if a player knows the sum (or alternatively her own integer), she must give the number. If neither player knows the sum, neither speaks. It is assumed that since the two players are candidates for entry into the Mathematics Club, they will be able to reason quickly and correctly. If the two integers written on the blackboard are 5 and 8, the maximum number of times that the bell will be rung is:

   (a) 1 (b) 3 (c) 4 (d) 5 (e) 8

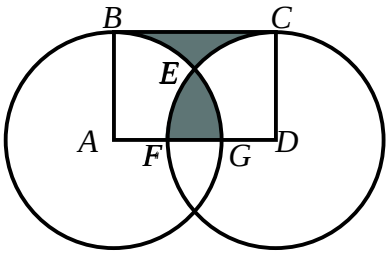
6. In a movie theater line, $x$ people are behind Mark, who is $y$ places in front of Sam. If there are $z$ people in front of Sam, how many people are in the line?

   (a) $z - x + y + 2$ (b) $z + x - y$ (c) $z - x + y - 1$ (d) $z + x - y + 1$ (e) $z - x + y$

7. Two numbers are such that their difference, their sum and their product are to one another as $1 : 7 : 18$. The product of the two numbers is:

   (a) 6 (b) 12 (c) 24 (d) 48 (e) none of these

8. In the diagram, congruent radii $\overline{AB}$ and $\overline{DC}$ intersect tangent $\overline{BC}$. If the shaded regions $BEC$ and $EFG$ have equal areas, and $\overline{AB} = 1$, find the area of quadrilateral $ABCD$.




(a) 1.25 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$ (e) $\frac{\pi}{6}$

9. The **prime** number 1999 can be written as $a^2 - b^2$ where $a$ and $b$ are integers. The value of $a^2 + b^2$ is:

(a) 1998000 (b) 1998001 (c) 1999000 (d) 3996001 (e) 3996000

10. In a river with a steady current, it takes Sam 6 minutes to swim upstream a certain distance, but only 3 minutes for her to swim back. Suppose, instead, she swam upstream the same distance as before but floated, not swam, back to her starting point. How long does this **round** trip take?

(a) 12 min (b) 15 min (c) 16 min (d) 18 min (e) 24 min

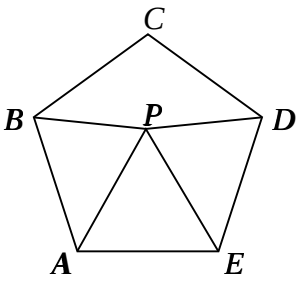
11. A real number $x$ is randomly chosen such that $-10 \leq x \leq 10$. What is the probability that $x$ is a solution to the inequality $x^2 - 6x + 8 \geq 0$?

(a) 0.1 (b) 0.3 (c) 0.5 (d) 0.7 (e) 0.9

12. The number of ordered pairs of integers which satisfy $(x + 2y)(2x + y) = 27$ is:

(a) 1 (b) 2 (c) 3 (d) 4 (e) more than 4

13. Given $ABCDE$ is a regular pentagon and triangle $APE$ is equilateral (as shown), find the measure of the obtuse angle $BPD$.

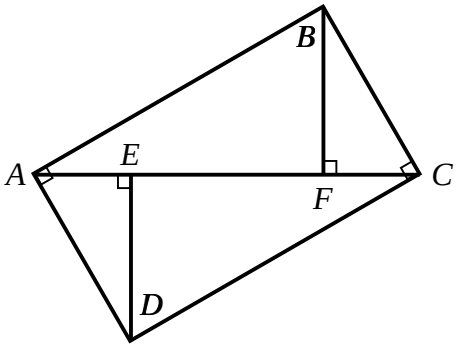



(a) 168° (b) 165° (c) 132° (d) 150° (e) none of these

14. Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. The probability that the sum of the two numbers on the balls removed is even is:

(a) $\frac{4}{9}$ (b) $\frac{9}{19}$ (c) $\frac{1}{2}$ (d) $\frac{10}{19}$ (e) $\frac{5}{9}$

15. $ABCD$ is a quadrilateral with right angles at $A$ and $C$. Points $E$ and $F$ are on the diagonal $AC$ such that $DE$ and $BF$ are both perpendicular to $AC$. If $AE = 3$, $DE = 5$, and $CE = 7$, then the length of $BF$ is:




(a) 3.6 (b) 4 (c) 4.2 (d) 4.5 (e) 4.8