BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest

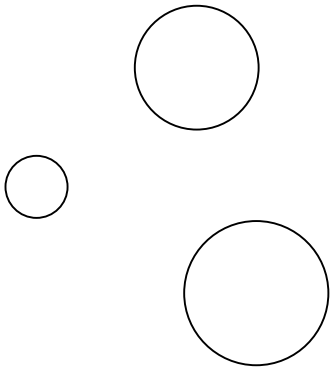
Part A Final Round April 30, 1999

1. Of 28 students taking at least one of Mathematics, English, or History, those students taking only one subject are all taking Mathematics. Of those students taking exactly two of the three subjects, it is known that the number taking Mathematics and English is identical to the number taking Mathematics only; six students are taking Mathematics and History; and the number taking History and English is even, non-zero, and equal to five times the number taking all three subjects. The number taking Mathematics and English only is:

   (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

2. How many circles can be drawn which are tangent to each of the 3 circles shown?

   (a) 2 (b) 3 (c) 5 (d) 6 (e) 8



3. A possible angle, x, in radians which satisfies $\log_2(\cos(x)) = -\frac{1}{2}$ is:

   (a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{4\pi}{3}$ (e) $\frac{7\pi}{4}$

4. Positive integers which read the same backwards as forwards are called *palindromes;* for example, 11, 252, and 31013 are palindromes. The number of palindromes less than 10^6, but greater than 10 is:

   (a) 999 (b) 10^3 (c) 1089 (d) 1989 (e) 2209

5. Five students are seated at a round table. After a break they are randomly assigned a seat at the same table. What is the probability that each student will have the same neighbours again? (It does not matter whether a given neighbour sits to the left or right.)

   (a) $\frac{1}{120}$ (b) $\frac{1}{60}$ (c) $\frac{1}{5}$ (d) $\frac{1}{10}$ (e) $\frac{1}{12}$

6. The squash season is nearing its end, and the current individual standings are shown in the chart. Each of the 8 players must still play 28 games, 4 with each of the other players. How many players still have a theoretical chance to at least tie for the championship?

Player:	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Games Won:	92	91	90	71	67	66	44	39
Games Lost:	48	49	50	69	73	74	96	101

   (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

7. If $1-y$ is used as an approximation to the value of $\frac{1}{1+y}$, $|y|<1$, then the absolute value of the ratio of the error made to the correct value is:

(a) y (b) y^2 (c) $\frac{1}{1+y}$ (d) $\frac{y}{1+y}$ (e) $\frac{y^2}{1+y}$

8. Of the following triangles given by the lengths of their sides, which one has the greatest area?

(a) 5, 12, 12 (b) 5, 12, 13 (c) 5, 12, 14 (d) 5, 12, 15 (e) 5, 12, 16

9. Determine the number of divisors of a positive integer m if

(i) m^2 has 35 divisors;

(ii) exactly two of the divisors of m are prime numbers.

(Note: 1 and the number itself are always counted as divisors. For example, the divisors of 12 are 1, 2, 3, 4, 6, 12, among which 2 and 3 are prime numbers. A *prime* number is a positive integer different from 1 that cannot be factored into a product of two smaller positive integers.)

(a) 12 (b) 7 (c) 5 (d) 10 (e) 8

10. At 3:00 the minute hand and the hour hand of a clock are perpendicular. At this time the second hand is aligned with the minute hand over the 12 on the clock. Call this position 0 seconds. At the next moment when the minute hand and the hour hand are again perpendicular, the position of the second hand is closest to the position:

(a) 8 seconds (b) 11 seconds (c) 43 seconds (d) 44 seconds (e) 48 seconds