BRITISH COLUMBIA COLLEGES

Junior High School Mathematics Contest

Part B – Final Round May 5, 2000

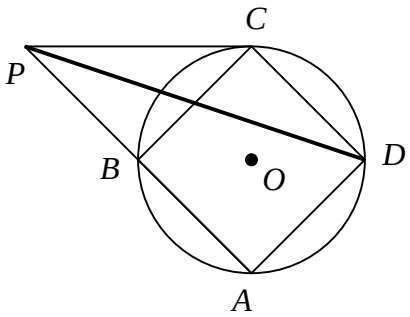
1. (a) How many 3-digit numbers can be formed using only the digits 1, 2, and 3 if both of the following conditions hold:

   (i) repetition is allowed;

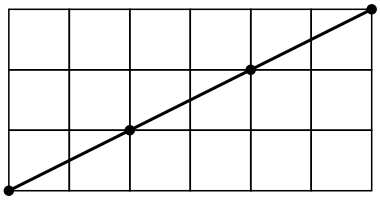
   (ii) no digit can have a larger digit to its left.

   (b) Repeat for a 4-digit number using the digits $1, 2, 3$, and 4.

2. The square $ABCD$ is inscribed in a circle of radius one unit. ABP is a straight line, PC is tangent to the circle. Find the length of PD. Make sure you explain thoroughly how you got **all** the things you used to find your solution!



3. If a diagonal is drawn in a 3×6 rectangle, it passes through four vertices of smaller squares. How many vertices does the diagonal of a 45×30 rectangle pass through?



4. Let a and b be any real numbers. Then $(a - b)$ is also a real number, and consequently $(a - b)^2 \geq 0$. Expanding gives $a^2 - 2ab + b^2 \geq 0$. If we add $2ab$ to both sides of the inequality, we get $a^2 + b^2 \geq 2ab$. Thus, for any real numbers a and b, we have $a^2 + b^2 \geq 2ab$.

   Prove that for any real numbers a, b, c, d,

   (a) $2abcd \leq b^2c^2 + a^2d^2$

   (b) $6abcd \leq a^2b^2 + a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 + c^2d^2$

5. A circular coin is placed on a table. Then identical coins are placed around it so that each coin touches the first coin and its other two neighbours.

   (a) If the outer coins have the same radius as the inner coin, show that there will be exactly 6 coins around the outside.

   (b) If the radius of all 7 coins is 1, find the total area of the spaces between the inner coin and the 6 outer coins.