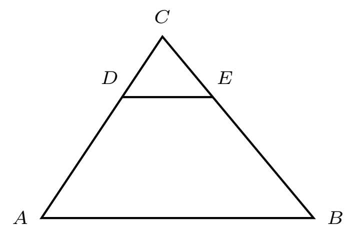
BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2002

Preliminary Round

March 6, 2002

1. In the diagram, sides AB and DE are parallel and $DE : AB = 1 : 3$. If the area of triangle CDE is 20, then the area of the trapezoid $DEBA$ is:



(a) 180 (b) 160 (c) 140

(d) 40 (e) 60

2. In the expansion of $\left(x^2 + \dfrac{1}{x^2}\right)^6$ there is one term that does not contain any x. This term has the value:

(a) 1 (b) 6 (c) 15 (d) 20 (e) 64

3. The number of different pairs of positive integers (m, n) that satisfy $m^n = 3^{20}$ is:

(a) 4 (b) 5 (c) 6 (d) 7 (e) more than 7

4. A line the sum of whose intercepts is N will be called an N-line. The sum of the y-intercepts of the two 3-lines which pass through the point $(-2, -4)$ is:

(a) -3 (b) -1 (c) 1 (d) 3 (e) 6

5. A calendar h centimetres long is designed with a picture a centimetres by b centimetres and a calendar pad c centimetres by d centimetres. The picture and the pad do not overlap and together they take up one-half of the area of the calendar. The width of the calendar is:

(a) $\dfrac{2ab + 2dc}{h}$ (b) $\dfrac{ab + dc}{2h}$ (c) $h(ab + dc)$ (d) $\dfrac{ab + dc}{h}$ (e) $2(ab + dc)$

6. One painter paints $5\,\mathrm{m}^2$ in 10 minutes, and a second painter paints $10\,\mathrm{m}^2$ in 5 minutes. If both painters are working together, then the number of minutes it will take them to paint $40\,\mathrm{m}^2$ is:

(a) 8 (b) 12 (c) 16 (d) 20 (e) 40

7. A prime number is an integer greater than one that is evenly divisible only by itself and one. The divisors of the year 2002 contain a sequence of three consecutive odd prime numbers. The number of years in the third millenium, i.e., between the years 2001 and 3000, that have this property is:

(a) 1 (b) 3 (c) 10 (d) 11 (e) 12

8. A certain brand of scotch tape has a thickness of 0.1 mm and forms a roll with a radius of 2 cm. The hollow plastic inner spool of the roll has a radius of 1 cm. The total length of the tape, in metres, is closest to:

(a) 1 (b) 9 (c) 20 (d) 25 (e) 100

9. A fair six-sided die, with faces numbered 1 to 6, is tossed two times. The probability that the two outcomes are in strictly ascending order, for example a 2 and then a 5, (equality is *not* allowed) is:

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$ (e) $\frac{5}{6}$

10. Two perpendicular lines L_1 and L_2 intersect at the point $Q\,(p, 2p)$ in the first quadrant. If $S\,(p-6, p)$ is on L_1 and $T\,(p+6, -p)$ is on L_2, then:

(a) Q may be any point on the line $y = 2x$
(b) there is no such point Q
(c) there is exactly one possible position for the point Q
(d) there are exactly two possible positions for the point Q
(e) the number of possible positions for the point Q is greater than two, but finite

11. The cost of making a rectangular table is calculated by adding two variables. The first is proportional to the area of the table and the other to the square of the length of the longer side. A 2×3-metre table costs \$50 to make and a 1.5×4-metre table costs \$64 to make. The cost of making a 2.5 metre square table, to the nearest cent, is:

(a) \$33.33 (b) \$45.83 (c) \$48.00 (d) \$50.00 (e) \$55.67

12. One of the chair lifts at Big White is three kilometres long, and a chair passes the starting point of the lift every 10 seconds. It takes a chair 6 minutes to reach the top of the lift. A skier gets on the lift just as another chair starts back down from the top. She arrives at the top of the lift just as a chair is starting up the mountain. The number of chairs on their way down that she passes on her trip up is:

(a) 36 (b) 37 (c) 71 (d) 72 (e) 73

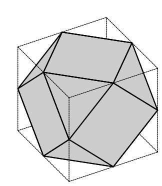
13. A square whose side length is r has a square inside it whose area is one-half the area of the larger square. There is a uniform border between the two squares. The width of the border is:

(a) $\frac{\sqrt{2}}{8}r$ (b) $\frac{2-\sqrt{2}}{2}r$ (c) $\frac{\sqrt{2}-1}{4}r$ (d) $\frac{4-\sqrt{2}}{16}r$ (e) $\frac{2-\sqrt{2}}{4}r$

14. Three circles with different diameters are mutually tangent externally. Their centres form a triangle with side lengths 5 cm, 6 cm, and 9 cm. The total area of the three circles is:

(a) 16π (b) 25π (c) 42π (d) 52π (e) 81π

15. A polyhedron is a solid bounded by plane (flat) surfaces only. These are the faces of the polyhedron. For example, a pyramid with a square base is a polyhedron with five faces, a pyramid with a triangular base is a polyhedron with four faces, and a cube is a polyhedron with six faces. For a pyramid with a flat base of area A and height h, where h is the perpendicular distance between the base and the vertex, the volume is $V = \frac{1}{3}Ah$.
The diagram shows a polyhedron constructed by cutting off all of the corners of a unit cube. The resulting polyhedron has faces that are squares or equilateral triangles all of whose sides have the same length. The volume of the resulting (shaded) polyhedron in the diagram is:



(a) $\frac{7}{8}$ (b) $\frac{5}{6}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$ (e) $\frac{1}{2}$