BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2009

Senior Preliminary

Wednesday March 4

Dedicated to the memory of Jim Totten, the inspiration for and co-founder of the BCSSMC

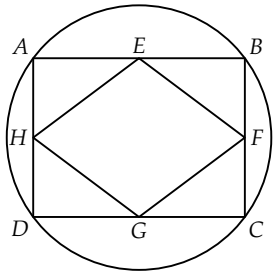
1. The expression

$$\frac{2010^2 + 2(2010)(2008) + 2008^2}{2010^2 - 2008^2}$$

equals:

(A) 4040100 (B) 4032064 (C) 45100 (D) 8407 (E) 2009

2. Quadrilateral $ABCD$ is inscribed in a circle. If $\overline{AB} = \overline{DC} = 8$ and $\overline{AD} = \overline{BC} = 6$, then the perimeter of the quadrilateral $EFGH$ formed by joining the midpoints E, F, G, and H of the sides AB, BC, CD, and DA is:



(A) 20 (B) 24 (C) 28

(D) 32 (E) 36

3. Jack walks up stairs one step at a time. Jill walks up stairs two steps at a time. Art, who likes to show-off, goes up three steps at a time. If each person starts with his or her left foot on the first step of the stairs, the first step on which all three will put their right foot is:

(A) 6 (B) 9 (C) 12 (D) 24 (E) Never happens

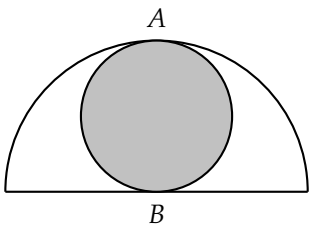
4. The value of the sum

$$1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{36} + \frac{1}{72} + \frac{1}{216} + \frac{1}{432} + \cdots$$

is:

(A) $\frac{3}{2}$ (B) $\frac{9}{5}$ (C) 2 (D) $\frac{9}{4}$ (E) 3

5. The shaded circle is tangent to the semicircle at the point A and the diameter of the semicircle at point B, the midpoint of the diameter of the semicircle. The ratio of the area of the shaded circle to the total area of the semicircle is:



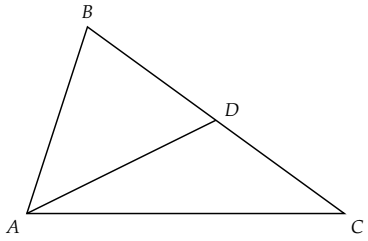
(A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{2}$

(D) $\frac{1}{3}$ (E) $\frac{1}{4}$

6. The area of the region enclosed by the graph of the equation $|x| + |y| = 4$ for $-4 \leq x \leq 4$ is:

(A) 16 (B) 24 (C) 32 (D) 48 (E) 64

7. In isosceles triangle ABC, sides AC and BC are equal. Point D lies on side BC such that both of the smaller triangles ABD and ACD are also isosceles. If AD and AB are equal, the measure of $\angle ABC$, in degrees, is:



(A) 70 (B) 64 (C) 60

(D) 80 (E) 72

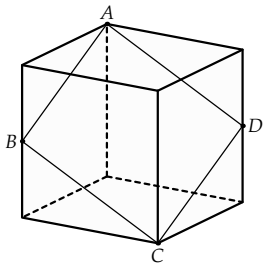
8. If $p^2 + \dfrac{1}{p^2} = 7$ and $p > 0$, then the value of $p + \dfrac{1}{p}$ is:

(A) 3 (B) $\frac{1}{2}\left(3 - \sqrt{5}\right)$ (C) 7 (D) 9 (E) $\sqrt{\frac{1}{2}\left(7 + 3\sqrt{9}\right)}$

9. There are 6 white socks and 10 red socks jumbled up in a box. If 2 socks are taken out at random, the probability of having a matched pair is:

(A) $\dfrac{1}{2}$ (B) $\dfrac{5}{9}$ (C) $\dfrac{3}{5}$ (D) $\dfrac{2}{3}$ (E) None of these

10. A cube with an edge length of 6 is cut by a plane to form a quadrilateral $ABCD$, where B and D are the midpoints of two edges of the cube. The area of the quadrilateral $ABCD$ is:



(A) 36 (B) $12\sqrt{6}$ (C) 45

(D) $18\sqrt{6}$ (E) 72

11. If $x = \sqrt{3\sqrt{2\sqrt{3\sqrt{2\cdots}}}}$, then the value of x^3 is

(A) 6 (B) 8 (C) 9 (D) 18 (E) Answer is infinite

12. A line contains the point $(3, 0)$ and is tangent in the first quadrant to the unit circle centred at the origin. The y-intercept of this line is:

(A) 1 (B) $\dfrac{3}{\sqrt{8}}$ (C) $\dfrac{9}{8}$ (D) $\dfrac{5}{4}$ (E) $\dfrac{\sqrt{8}}{3}$