BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2006

Senior Final Round, Part B

Friday May 5, 2006

- 1. Determine the number of sequences of consecutive integers whose sum is 100.
- 2. A cubical glass tank with sides of length one metre is placed on a horizontal table and half filled with water. Thus, the depth of the water in the tank (the distance of the surface of the water from the surface of the table) is one half metre. The tank is rotated about one of the edges that is on the table so that one face of the tank makes a 30° angle with the table. Find the depth of the water in the tank after the rotation.
- 3. The lengths of the sides of a triangle are 13, 13, and 10. The *inscribed circle* of this triangle is the circle with centre inside the triangle that is tangent to each of the three sides of the triangle. See the diagram. Find the radius of the inscribed circle.



4. Five positive integers a, b, c, d, and e greater than one make the following conditions true:

a (b + c + d + e) = 128 b (a + c + d + e) = 155 c (a + b + d + e) = 203 d (a + b + c + e) = 243e (a + b + c + d) = 275

Find the five integers.

5. A full binary tree consists of a root **node** which has two **children**, a right child node and a left child node, and each child node has two children, until the top of the tree is reached where each node has no children. In a certain full binary tree each node is numbered, starting with one at the root, numbering from left to right across each level. The diagram shows the first four levels of such a tree. The root of the tree is placed at the origin of an *xy*-coordinate system, with the *x*-axis horizontal and the *y*-axis vertical, as shown. If the spacing between the levels of the tree is 2 units in the *y*-direction and the spacing between the nodes



at the top level that contains the node numbered 2006 is 2 x-units, find the coordinates of the node numbered 2006.