BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2005

Final Round, Part B

Friday May 6, 2005

1. The digits 1, 2, 3, 4, and 5 are each used once to compose a five digit number $abcde$ such that the three digit number abc is divisible by 4, bcd is divisible by 5, and cde is divisible by 3. Find the digit a.

2. An urn contains three white, six red, and four black balls.

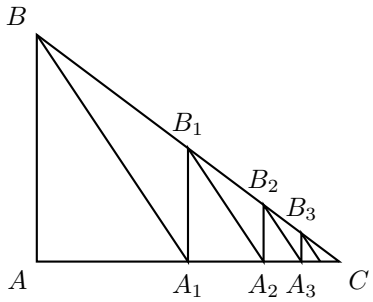
   (a) If one ball is selected at random, what is the probability that the ball selected is red?

   (b) If two balls are selected at random, what is the probability that they are both black?

   (c) If two balls are selected at random, what is the probability that they are both black, given that they are the same colour?

3. In the diagram ABC is a right triangle with $\overline{AB} = 3$ and $\overline{AC} = 4$. Further, each line segment A_iB_i is perpendicular to AC, A_1 bisects AC, and A_{i+1} bisects A_iC. Find the total length of the sequence of diagonal segments

$$\overline{BA}_1 + \overline{B_1A}_2 + \overline{B_2A}_3 + \cdots$$



4. The equation

$$x^2 - 3x + q = 0$$

has two real roots α and β. Knowing that $\alpha^3 + \beta^3 = 81$, find the value of q. Hint: It is not best to use the quadratic formula.

5. A four-digit number which is a perfect square is created by writing Anne's age in years followed by Tom's age in years. Similarly, in 31 years, their ages in the same order will again form a four-digit perfect square. Determine the present ages of Anne and Tom.