BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2004
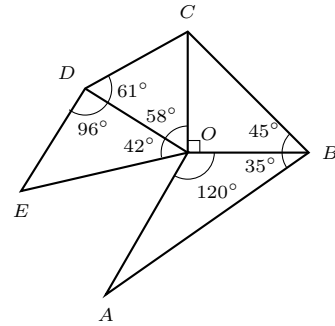
Final Round, Part A

Friday May 7, 2004

1. A number is called a palindrome if the number is unchanged when its digits are reversed. Hence, 8338 and 57275 are palindromes. The number of 4-digit palindromes is:

   (a) 80   (b) 90   (c) 100   (d) 110   (e) none of these

2. In the diagram at the right, the shortest segment is:

   (a) OB (b) OC (c) OD

   (d) CD (e) DE

3. Each of the integers 1 to 9 is written on a different slip of paper, and all nine slips of paper are placed in a jar. You pick one of the slips at random, record the number and return the slip to the jar. You pick a second slip from the jar. The digit which is most likely to be the units digit of the <u>sum</u> of the two numbers that you picked is:

   (a) 0   (b) 1   (c) 5   (d) 9   (e) all digits equally likely

4. If $f(3x) = \dfrac{3}{1+x}$ for all x not equal to -1, then $3f(x)$ is equal to:

   (a) $\dfrac{9}{1+x}$ (b) $\dfrac{9}{3+x}$ (c) $\dfrac{27}{3+x}$ (d) $\dfrac{1}{3+x}$ (e) $\dfrac{9}{1+3x}$

5. A fair 6-sided die, with sides numbered from 1 to 6, is tossed two times. The probability that the two outcomes will be in strictly increasing order is:

   (a) $\dfrac{5}{12}$ (b) $\dfrac{13}{18}$ (c) $\dfrac{11}{36}$ (d) $\dfrac{2}{3}$ (e) none of these

6. The Fibonacci sequence begins: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$. (Each number beyond the second number is the sum of the previous two numbers.) The notation f_4 means the fourth number; for example $f_4 = 3$ and $f_7 = 13$. A term of the Fibonacci sequence that is divisible by 3 is:

   (a) f_{49} (b) f_{75} (c) f_{196} (d) f_{379} (e) f_{999}

7. Bob bought twelve postage stamps in a sheet of 3×4 stamps, as shown. Meg asks him to give her four stamps that are all joined together, with each stamp joined to the other three along at least one edge. The number of ways of separating four stamps this way is:

1	2	3	4
5	6	7	8
9	10	11	12

(a)   45   (b)   51   (c)   65

(d)   100   (e)   220

8. A triangle has sides of lengths 10, 24, and n, where n is a positive integer. The number of values of n for which this triangle has three acute angles is:

(a)   1   (b)   3   (c)   4   (d)   5   (e)   more than 5

9. The number of sets of two or more consecutive positive integers that have a sum of 105 is:

(a)   6   (b)   7   (c)   8   (d)   9   (e)   10

10. The centres of four circles of radius 12 form a square. Each circle is tangent to the two circles whose centres are the vertices of the square that are adjacent to the centre of the circle. A smaller circle, with centre at the intersection of the diagonals of the square, is tangent to each of the four larger circles. The radius of the smaller circle is:

(a) $12\sqrt{2}$ (b) $14 + 13\sqrt{2}$ (c) $12\left(\sqrt{2} + 1\right)$

(d) $12\left(\sqrt{2} - 1\right)$ (e) $\dfrac{12}{\sqrt{2}}$