BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2003

Final Round, Part A

Friday May 2, 2003

1. The area of the region bounded <u>below</u> by the graph of the equation $x^2 + y^2 = 4$ and <u>above</u> by the graph of the equation $y = -|x| + 2$ is:

   (a) $2\pi - 4$ (b) $2\pi + 4$ (c) $4\pi - 4$ (d) 4π (e) $4\pi + 4$

2. Assuming that a, b, c, and d are distinct prime numbers greater than 10, the number of common multiples of $14a^7b^5c^4$ and $98a^3b^{15}d^7$ which are factors of the product of the two numbers is:

   (a) 15 (b) 96 (c) 140 (d) 588 (e) 729

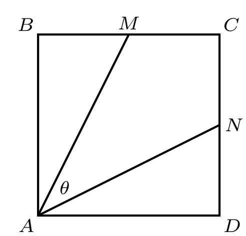
3. The sides of a regular polygon with n sides ($n > 4$) are extended to form a star. The number of degrees in the angle at each point of the star is:

   (a) $\dfrac{180}{n}$ (b) $\dfrac{360}{n}$ (c) $180 - \dfrac{90}{n}$ (d) $\dfrac{180}{n}(n-2)$ (e) $\dfrac{180}{n}(n-4)$

4. Let m and n be the roots of the equation $ax^2 + bx + c = 0$. Let $px^2 + qx + r = 0$ be a quadratic equation for which $m + 2$ and $n + 2$ are roots. If $p = a$, then $q + r$, expressed in terms of a, b, and c is:

   (a) $c + 3b$ (b) $c - b$ (c) $c + 3b + 8a$ (d) $c - b + 4a$ (e) $c - b + 8a$

5. In the diagram, $ABCD$ is a square with side length 2. Let M and N be the midpoints of the sides BC and CD, respectively, and let θ be the angle $\angle MAN$. The value of $\cos\theta$ is:



   (a) $\dfrac{\sqrt{5}}{5}$ (b) $\dfrac{\sqrt{10}}{5}$ (c) $\dfrac{3}{5}$

   (d) $\dfrac{4}{5}$ (e) $\dfrac{\sqrt{3}}{2}$

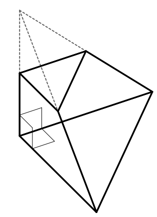
6. Suppose that the domain of the function $f :\mapsto f(x)$ is the set $\mathcal{D}$, and the range is the set $\mathcal{R}$, where $\mathcal{D}$ and $\mathcal{R}$ are subsets of the real numbers $\mathbb{R}$. Consider the functions

$$f(2x),\ f(x+2),\ 2f(x),\ f\left(\frac{x}{2}\right),\ \frac{f(x)}{2},\ f(x+2) - 2$$

   If m is the number of the functions listed above that must have the same domain as f, and n is the number that must have the same range as f, then the ordered pair (m, n) is:

   (a) $(0, 5)$ (b) $(1, 5)$ (c) $(2, 3)$ (d) $(3, 2)$ (e) $(3, 3)$

7. A tetrahedron, a solid with four triangular faces, is cut from the corner of a unit cube. It has three faces that are mutually perpendicular isosceles right triangles and the fourth face an equilateral triangle. The three perpendicular edges are edges of the cube and so have unit length. A plane parallel to one of the isosceles triangles bisects the third perpendicular edge to form the solid shown. The total surface area of the solid is:



(a) $\dfrac{3+\sqrt{3}}{4}$ (b) $\dfrac{9+3\sqrt{3}}{8}$ (c) $\dfrac{11+3\sqrt{3}}{8}$

(d) $\dfrac{17}{8}$ (e) $\dfrac{5}{2}$

8. The odd positive integers are arranged in the pattern indicated. The value of the entry in the 43^{rd} row and 29^{th} column is:

1			
3	5		
7	9	11	
13	15	17	19
21		...	

(a) 1863 (b) 1951 (c) 2003 (d) 2005 (e) 2501

9. Three balls marked 1, 2, and 3, respectively, are placed in an urn. One ball is drawn from the urn, its number is recorded and the ball is replaced in the urn. This process is repeated a second and a third time. Each ball is equally likely to be drawn on each occasion. If the sum of the numbers drawn is 6, then the probability that the ball numbered 2 was drawn all three times is:

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{7}$ (d) $\dfrac{1}{8}$ (e) $\dfrac{1}{27}$

10. A point P is marked on the rim of a right-circular cone with radius 1 and slant height 3. The length of the shortest path that can be drawn on the surface of the cone from P back to P and that goes around the cone once is:

(a) 3 (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{3\sqrt{3}}{2}$ (d) $3\sqrt{3}$ (e) 2π