BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2006

Senior Final Round, Part A

Friday May 5, 2006

1. If one stamp is randomly selected from a 12×12 sheet of stamps, then the probability that the stamp selected is one of the stamps on the border of the sheet is:

   (A) $\dfrac{1}{12}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{11}{36}$ (D) $\dfrac{25}{36}$ (E) $\dfrac{2}{3}$

2. Consider the set of integers $\{1, 2, 3, 4, 5, 6, 7, 8\}$. The number of ways in which exactly five of these numbers can be selected, without repeating any number, so that the sum of the numbers selected is at least 20 is:

   (A) 43   (B) 45   (C) 49   (D) 54   (E) 56

3. It takes 10 minutes for a canoeist to paddle perpendicular to the current across a river of uniform width. (Of course, while crossing the river the canoeist drifts downstream.) The canoeist then paddles upstream for 50 minutes, recrosses the river and paddles downstream for 20 minutes, returning to the starting point. The ratio of the canoeists speed in still water to the speed of the river current is:

   (A) $3 : 1$ (B) $2 : 1$ (C) $1 : 2$ (D) $3 : 2$ (E) $1 : 3$

4. Recall that when light is reflected off a mirrored surface, the angle of incidence equals the angle of reflection. Two mirrored surfaces are placed at an angle of $20°$ and a ray of light is incident from the right. See the diagram. The maximum number of reflections possible is:
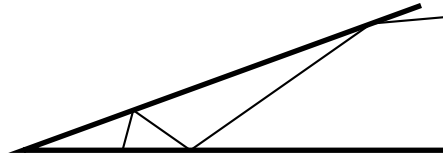
   (A) 4   (B) 5   (C) 7   (D) 8   (E) infinite

5. In the circle shown line segment CD is perpendicular to the chord AB. Further, $\overline{AC} = 4$, $\overline{CD} = 6$, and $\overline{BC} = 17$. The radius of the circle is:

   (A) $\dfrac{65}{6}$ (B) 11   (C) $\dfrac{21}{2}$

   (D) $\dfrac{\sqrt{513}}{2}$ (E) 13
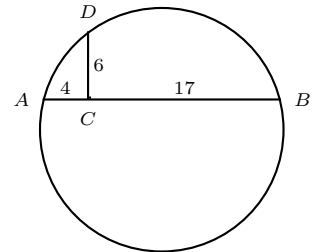
6. Six points are placed on the circumference of a circle and a line is drawn between each pair of points. The maximum possible number of regions that lines form inside the circle is:

   (A) 16   (B) 28   (C) 29   (D) 31   (E) 33

7. In the 2005 Grammy Awards there were 103 different categories and five artists were nominated in each category. Some artists were nominated in more than one category. There were 50 artists nominated in at least two categories, 35 nominated in at least three categories, 24 nominated in at least four categories, and 12 nominated in five categories. No artist was nominated in more than five categories. The number of distinct artists nominated for at least one Grammy Award in 2005 was:

   (A) 154   (B) 280   (C) 345   (D) 361   (E) 394

8. A circle of radius 1 has centre near one vertex of a square
    in such a way that $\overline{AB} = \overline{BC} = a$, as shown. The value
    of a for which the distance $\overline{OB}$ equals $\frac{1}{2}$ is:
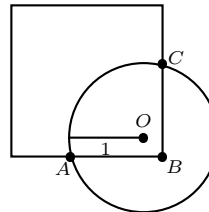
    (A) $\dfrac{\sqrt{3}-1}{2}$ (B) $\dfrac{\sqrt{7}+1}{2\sqrt{2}}$ (C) $\dfrac{\sqrt{3}+1}{2}$

    (D) $\dfrac{\sqrt{7}+1}{\sqrt{2}}$ (E) $\dfrac{\sqrt{7}-1}{2\sqrt{2}}$

9. When you see an analog clock for sale, the minute and hour hand are usually positioned so that they
    are the same distance from the six on the clock, at roughly 8:20. For the time closest to 8:20 when the
    minute hand and the hour hand are an equal angular distance on either side of the six, the number of
    seconds from a time with an exact minute, to the nearest second, is:

    (A) 27   (B) 28   (C) 29   (D) 32   (E) 33

10. Six balls numbered 2, 3, 4, 5, 6, and 7 are placed in a hat. Each ball is equally likely to be chosen. If
    two balls are chosen, without replacement, the probability that the sum of the numbers on the selected
    balls is a prime number is:

    (A) $\dfrac{4}{15}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{7}{15}$ (E) $\dfrac{2}{3}$