BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2009

Senior Final, Part B

Friday May 8

Dedicated to the memory of Jim Totten, the inspiration for and co-founder of the BCSSMC

1. Of the students in a class 17 can ride a bicycle, 13 can swim, and 8 can ski. No student is able to perform all three of these activities. All the bicyclists, swimmers, and skiers have received a grade of C or better in the class. Six students in the class received a grade less than C. Determine the smallest possible number of students in the class.

2. (a) Use the fact that the last two digits of 3^9 are 83 to determine the last two digits of 3^{10}.
   (b) Determine the last two digits of 3^{20}.
   (c) Determine the last two digits of 3^{2009}.

3. Consider the equation
$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$$
where x, y, and z are positive integers.
   (a) Find all of the solutions to the equation above for which $x \leq y \leq z$.
   (b) Determine the total number of solutions to the equation above with the restrictions from part (a) removed.

4. Three 1's, three 0's, and three -1's are placed in the 3×3 grid shown in such a way that the sign, or value if it is zero, of the sum in each row, each column, and one diagonal is as shown. The value of the sum of the numbers along the other diagonal must be an integer between -3 and 3, inclusive.

			+
			−
			0
0	+	−	0

   (a) Draw one grid, with numbers filled in according to the conditions given, for which the sum along the other diagonal is 3.
   (b) Draw one grid, with numbers filled in according to the conditions given, for which the sum along the other diagonal is 2.
   (c) Determine which of the values 1, 0, and -1 can be obtained as the sum along the other diagonal. For any value that is possible, draw one grid, with numbers filled in according to the conditions given, for which the sum along the other diagonal is that value. For any value that is impossible, demonstrate that the value is impossible.

5. Find the area of the region inside the circle $x^2+y^2=6x$ but outside the circle $x^2+y^2=27$.